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Water table frequencies of tile-drained land based on moisture balance

Hugh Oswell Vaigneur
Iowa State University

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WATER TABLE FREQUENCIES OF TILE-DRAINED LAND

BASED ON MOISTURE BALANCE

by

Hugh Oswell Vaigneur

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Head of Major Department

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Dean of Graduate College

Iowa State University
Of Science and Technology
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1965

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INTRODUCTION AND OBJECTIVES

Introduction

Precipitation falling on the soil may be allocated to evaporation, runoff or deep seepage, or it may be stored in the soil for future use. The behavior of water after it falls onto the land has been the subject of intense study during recent years. Theoretical determinations of fundamental relationships pertaining to flow through soils have been developed, and compared to experimentation in many cases. Plant-climate relationships have been investigated, giving answers to many pertinent questions. Yet, due to the extensive variability of certain factors in practice, there is an apparent need for continued searching of various aspects of soil-hydrology relations which may yield means of predicting the time and duration of excessive moisture.

It has been customary to divide field drainage problems into two broad groups, according to whether surface water or subsurface water was the main problem. This problem was primarily concerned with subsurface water for the case when precipitation was responsible for maintaining the water table. It was desired to establish certain design criteria relating soil moisture conditions to drainage requirements, which would provide an adequate root environment for crop plants and facilitate timely tillage operations.

In some cases, nature has provided a desirable balance between available moisture for plants and the plant moisture requirements. In other cases the supply of moisture has been either inadequate or excessive, which could have been the result of amount or distribution of precipitation, or a combination of these factors.

When artificial drainage was used to remove moisture excess, it was pertinent to evaluate certain aspects of hydrology, soil-moisture characteristics and hydraulics in order to determine the depth, spacing, and size of conduits to convey the water being removed. It was common to assume a steady-state condition and design the drainage system such that a predetermined elevation of the water table could be retained under a given rainfall. This procedure was satisfactory in assuring adequate drainage for the design rainfall. Steady-state conditions for which a number of theoretical solutions exist are encountered for such short durations that the selection of this condition for design criteria may not be consistent with economic demands. Likewise, the transient analysis where the water table is followed down after one saturation, or after a series of saturations at regular intervals, may not adequately lend itself to design problems where a calculated risk is acceptable at a predetermined recurrence level. There is a need for knowledge on what happens when erratic, variable precipitation occurs during a specified season with a known artificial drainage installation. Additional consideration should be given to the degree of drainage, as this problem is related to economic and other factors that control the practicality of removing excessive moisture from agricultural soils.

Frequently, necessity has demanded that drainage systems be designed on the basis of experience with other systems in similar soils. This in general has resulted in designs with lateral spacing in the vicinity of 100 feet. From an economic standpoint this has been justified where poor natural drainage was accompanied by fertile soils. However, for design problems where the feasibility of artificial drainage hinges on an economic evaluation, it would be advantageous to have available a procedure related to the expected distribution of water table fluctuations.

The water table was used as a criterion for evaluating the state of drainage, though it is recognized that the water table is only a pressure contour, and other factors, such as the height of the capillary fringe, are important when evaluating the moisture content of the soil between the ground surface and the water table. From this standpoint, the scope of the problem was limited to transient water table behavior where the flow of water in an unsaturated zone was specifically excluded. This assumption was justified on the basis of the water balance which produced no effect on the water table until the available water-holding capacity of the soil was satisfied.

The problem was further reduced by specifying the critical period for drainage as the three month interval including April, May, and June. This selection was based on the fact that the frost leaves the ground late in March or early in April and is followed by wet conditions due to snow melt and spring rains. The average rainfall during this period is 11.38 inches, and the use of water by plants is very limited. The last of June was selected as the end of the critical period on the basis of weather data and the extent of the development of a crop in central Iowa. This assumption was supported by the hydrograph[1] of the water table shown in Figure 1. This graph was obtained from water table measurements under meadow in a Webster Silt Loam soil where the impermeable layer was about 10 feet below the ground surface. The problem was slanted toward the production of continuous corn in central Iowa on gently sloping soil with a root zone of 5 feet and an available water-holding capacity of 9 inches. It was assumed that an impervious layer below the root zone prohibited deep percolation, therefore requiring artificial drainage when the precipitation produced more water than could be stored in the root zone.

[1]Data from files of Agricultural Engineering Department, Iowa State University of Science and Technology, Ames, Iowa.

Objectives

The specific objectives were as follows:

1. To determine the portion of total precipitation (excess) which contributed to subsurface drainage demands.

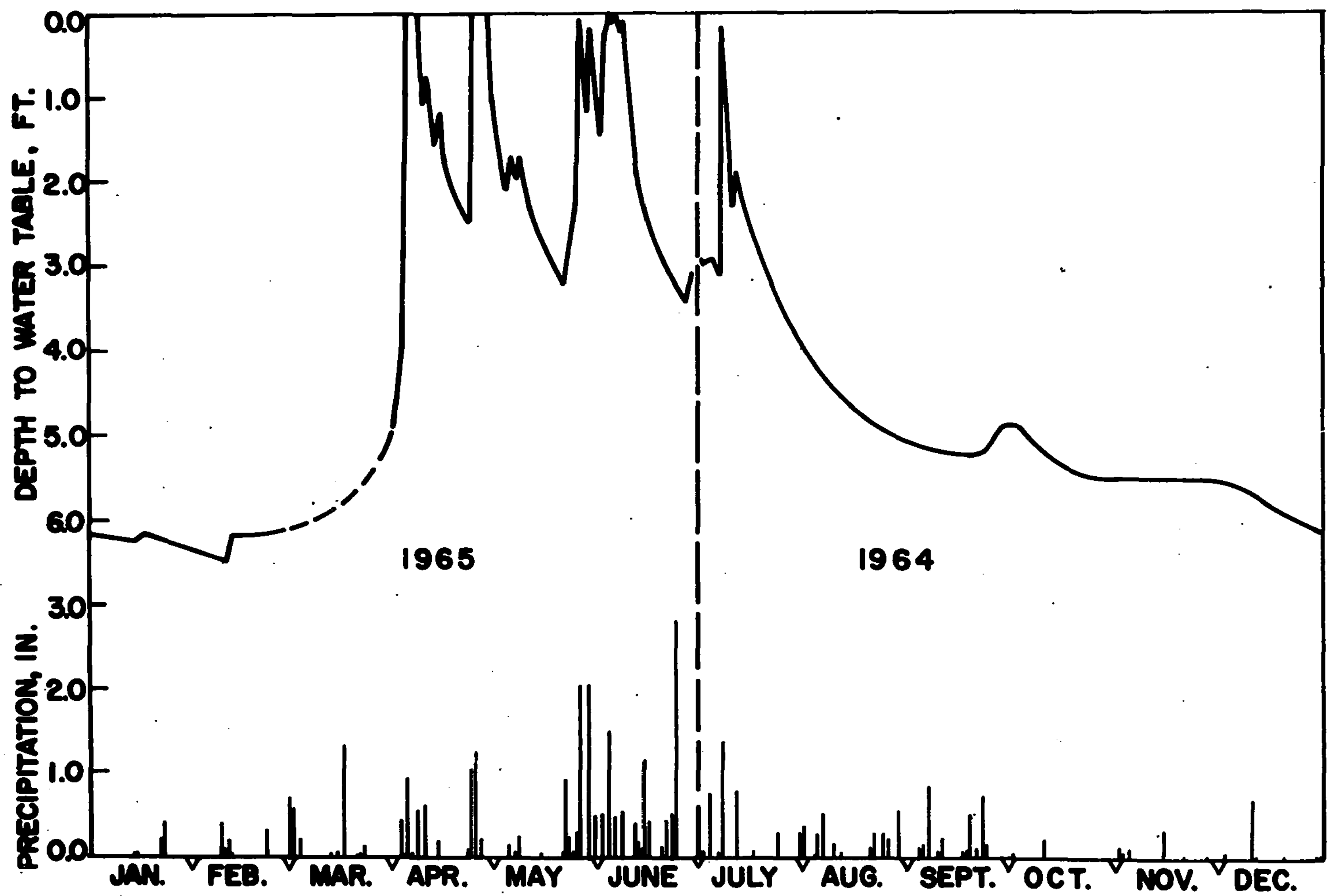
2. To compare the water table behavior obtained from observations in a viscous-fluid model with that calculated from theoretical developments.

3. To make a frequency distribution of water table levels calculated by the use of the excess derived from a water balance.

4. To compare the effect of the position of the impervious layer on the water table behavior.

5. To evaluate the height of the water table at which time the given tile spacing would be capable of collecting more water than the main lines could remove for a designated drainage coefficient.

Figure 1. Hydrograph of water table under meadow in a Webster Silt Loam soil from July 1, 1964 to June 30, 1965


DEPTH TO WATER TABLE, FT.
0.0
1.0
2.0
3.0
4.0
5.0
6.0
PRECIPITATION, IN.
3.0
2.0
1.0
0.0
1965
1964
JAN.
FEB.
MAR.
APR.
MAY
JUNE
JULY
AUG.
SEPT.
OCT.
NOV.
DEC.

REVIEW OF LITERATURE

Drainage Requirements Related to Spacing of Laterals

Tile drainage is by far the most widely accepted method of artificial drainage. Subsurface drains either singly or in combination may be required for an adequate water disposal system. The principle types of materials for subsurface drains are clay tile and concrete tile. Other materials have been investigated, but have received minor acceptance compared to the clay and concrete materials. For uniform drainage of large areas, the gridiron and herringbone layouts are common, and combinations of these two systems may be adapted for local conditions. In considering these systems, the question of depth and distance between laterals is of major importance from the standpoint of satisfactory functional requirements and from an economic point of view. The Iowa Drainage Guide (25) recommends, for a depth of 4 feet, a spacing of between 70 and 150 feet, depending upon location and soil-moisture characteristics.

Drainage coefficient

Tile drain spacings are generally based on a drainage coefficient which is defined as the depth of water to be removed from the drainage area in a unit of time, commonly designated as inches of water per 24 hours (20). The coefficient is usually based on past experience with similar soil, crop, and climatic conditions. Selection of a drainage coefficient does not necessarily indicate that this amount of water will be able to move through the soil to the drains, but it does indicate the capacity for which the drainage system has been designed.

Lynde (45) in North Carolina measured the outflow from tile and recommended a drainage coefficient of 1/4 inch for drain spacings of 100 feet or more and 3/8 inch for drains closer than 100 feet. Schlick (55) in Iowa made similar studies from large drainage systems. He recommended drainage coefficients from 5/16 to 3/8 inch for spacings over 100 feet and 1/2 inch or more where spacings are decreased to 50 feet. Recommended coefficients for various areas of the United States are given by Frevert *et al.* (20).

Soil characteristics

Beauchamp and Fasken (4) emphasized the role of soil types in the design of subsurface drainage systems. Baver (3) pointed out that the need for drainage was related to the air capacity of the soil, which in turn was a function of the non-capillary porosity. Russell (53), in a review of research in soil drainage up to about 1930, stated that field trials of various drain spacings were evaluated according to the degree of crop production. Then the spacings were correlated with physical properties of the soil such as hygroscopicity, percentage of particles less than 20 microns in size, and heat of wetting. By applying this method, the entire determination was based on the results of a single year, no account being taken of climatic and soil structure factors. Accordingly, it was not possible to apply the relations directly to other regions where precipitation and evaporation were different from those in the region investigated.

Neal (50) found that drain spacings could be determined from the plastic limits and the clay content. Neal's work was not based on soil conductivity, as are most recent methods, because field techniques of conductivity measurements were not developed at that time. Neal also reported that crops were not seriously injured if the water table was held at least 0.5 foot below the surface, and was lowered 0.5 foot during the first 12-hour period, and another 0.5 foot during the next 17 hours.

Drainage theory

More recently, a number of analytical solutions based on soil conductivity have been developed for lateral spacings. Theoretical solutions are based on either a stationary water table, where the flow supplied by percolation through the surface is equal to the flow removed by the tile, or on a falling water table which is time dependent (56).

It may be further stated that theoretical solutions generally depend on either the Dupuit-Forchheimer assumptions or on potential theory (6). The Dupuit-Forchheimer assumptions (68) are: (a) all streamlines in a system of gravity flow towards a shallow sink are horizontal; and (b) the velocity along these streamlines is proportional to the slope of the free water surface but independent of the depth. Potential theory requires that the velocity potential in a flow regime satisfy Laplace's equation (54).

Because of short, high intensity storms and large fluctuations of the water table in humid regions, a static water table is seldom encountered. However, steady-state conditions may be approached in areas where a constant low rate of rainfall prevails for a relatively long period. Kirkham and De Zeeuw (30) reported rainfall and water table data from a drainage experiment in The Netherlands which provided nearly steady-state conditions.

Schwab (56) suggested several spacing formulas based on a static water table including the work of Aronovinci and Donnan (1), van Deemter (66), Hooghoudt (24) and Visser (73).

Kirkham (27) derived theoretical formulas for the height of all points of an arched water table existing under conditions of steady rainfall seeping into homogeneous soil drained by tubes or ditches. In later work, Kirkham (28) modified the earlier equations for water-table height to take into account the head loss in the region lying below the water table but above the plane of the water level in the drains. Toksöz and Kirkham (63) presented in graphical and tabulated form a drain spacing formula which was based on Kirkham's rigorous mathematical developments.

Spacing formulas based on a falling water table have been reported by several investigators such as Walker (74) and Dumm (15). Van Schilfgaarde et al. (71) found, from a theoretical standpoint, that Walker's equation would result in too large a spacing. The spacing formula developed by Glover, and reported by Dumm (15), is known as the Glover formula. Equations for the transient water table were frequently developed by integrating a differential equation derived from a steady-state condition.

Dylla (18) proposed a solution to the drain spacing problem by using a set of prime drawdown curves for various positions of the impermeable layer with respect to the drain tile. This work was based on a transient flow equation obtained by integrating a form of the Donnan steady-flow equation.

Kirkham and Gaskell (29) assumed that the falling water table could be treated as a succession of steady-state positions. It was assumed also that there existed a constant pore space which was entirely drained at the instant the water table passed. Van Schilfgaarde (69) found that Kirkham and Gaskell's relaxation procedure gave results on the behavior of the changing water table comparable to that obtained by the Glover equation.

Van Schilfgaarde (67) developed an equation which expressed the drain spacing as a function of the geometry, the soil characteristics, and time. The resulting equation was based on the Dupuit-Forchheimer assumptions but did not assume a constant thickness of the water-bearing stratum. It did provide a correction for the convergence of flow lines in the vicinity of the drains.

Bouwer and van Schilfgaarde (7) presented a simplified procedure for predicting rate of fall of the water table in tile-drained land. The procedure was based on steady-state theory and abrupt drainage of pore space. The apparent accuracy of the design procedure was discussed and compared with other solutions.

Kirkham (26) used two physical artifices to simplify the mathematics of the falling water table. It was assumed that precipitation seeped vertically downward, and without loss of head due to friction, to the level of the drain centers. The resulting formulas were compared with field data and found to be adequate in predicting the fall of the water table for a given drain spacing. The theory was not based on the Dupuit-Forchheimer methods, and convergence effects were taken into account.

Luthin (43) developed a spacing equation based on the assumption that the rate of flow into a tile line was directly proportional to the height of the water table above the drain at the midpoint between tile lines, and to the soil hydraulic conductivity. In addition to the hydraulic conductivity, the drainable pore space expressed as a function of soil moisture tension was necessary.

Maasland (46) worked with the problem of water table fluctuations in response to intermittent instantaneous recharge. Maasland suggested that recurrent rainfall in humid regions may produce an effect similar to that of the recharge patterns used in his investigation. The results of the study were stated in infinite series which were applicable to any number of successive recharges. Glover's equation (15) was compared to Maasland's results and it was shown that the former equation was somewhat inadequate for an analysis of the fluctuation of the water table in response to a succession of instantaneous recharges.

Dumm and Winger (17) presented examples of the use of curves developed by the Bureau of Reclamation for drain-spacing design. The curves were based on mathematical treatment of the transient-flow concept. The procedure involved a design scheme which permitted the annual discharge to be equal to the annual recharge. The condition was defined as "dynamic equilibrium" when the highest level and the range of water table fluctuation became reasonably constant from year to year.

Talsma and Haskew (60) used data from selected farm tile drainage systems in Australia to compare the physical response of water tables to that predicted by several theories. It was concluded that Hooghoudt's theory (24) was adequately supported when flow boundaries were sharply defined. The field data also supported Kirkham's analysis (27) where the physical assumption underlying that analysis was reasonably met. Field data on the rate of lowering of the water table generally supported Glover's analysis (15), although some caution appeared to be necessary for design in cases where there was an impermeable layer at a small distance below the tile.

Luthin and Worstell (44) used a refinement in theory which took into consideration the effect of soil-moisture tension on the amount of water drained out of the soil. They found that when soils had an impervious layer more than 2 feet below the drains, the rate of flow into a tile line had a linear relationship with the water table height at the midpoint between the drains. Where the impervious layer was closer than 2 feet, the relationship was no longer linear.

Dumm (16) discussed a drain-spacing equation which involved a fourth-degree parabola to represent the initial water table conditions for the case when the drains were above the barrier. The use of a parabolic shape was considered to give better agreement with the shape of the water table, than the flat water table formerly used by Dumm (15). It was shown that correction for convergence could be made by either of two methods.

Ligon et al. (41) showed how the steady-state water table solution of Kirkham (27, 28) could be extended to the falling water table case for the particular problem of flow to parallel open-ditch drains partially filled with water. A graphical procedure was developed such that the results obtained from the falling water table equations could be compared to results obtained from a glass bead-glycerol model (37) of the drainage system. It was found that the theory was valid to within a relatively small error.

Grover and Kirkham (22) used a glass bead-glycerol model to develop curves relating lateral spacings to a dimensionless term involving time, hydraulic conductivity, drainable porosity, and depth to the water table below the ground surface at a point midway between drains. Several examples of field problems were worked out in numerical detail illustrating the required spacing necessary to secure a given drawdown in a designated interval of time.

Asseed (2) studied the falling water table in a glass bead model after steady-state conditions were terminated. A series of treatments were used where the location of the impervious layer and the depth of tile were varied. The effect of the impervious layer agreed with former findings in that there was a critical depth of the layer such that greater depths did not appreciably affect the rate of fall of the water table. The critical depth was dependent upon the geometry of the flow region.

Ede (19) has proposed a scheme for the assessment and design of field drainage systems on a hydrologic basis. The general approach was to select the most severe drainage period of about one month. It was assumed that all of the precipitation entered the ground except for loss due to evapotranspiration, which was negligible during the winter season. Ede used a combination of moisture balance, empirical determination of tile discharge, and measured water table behavior to obtain limits on the water table for a specified drain spacing. The objective was to determine tile spacings such that the water table could be retained within a designated fluctuation margin.

Wesseling (75) presented a method for evaluating the adequacy of an existing tile-drainage system. A linear relationship between tile discharge and height of the water table above the tile line was assumed. The general procedure involved establishing a mean precipitation rate which was incorporated into an equation for determining the rainfall on any specified day. The final analysis was based on climatic factors which involved the frequency of a given water table for a specified drainage design.

Kraijenhoff (36a) used a linearization technique on the differential equation resulting from the Dupuit-Forchheimer assumptions to obtain the following equation:

$$Y_t = (4j)/(f\Pi)\ P_t \sum_{n=1,-3,5\ldots} (1/n^3)\ [1-\exp(n^2 t/j)] \qquad 1$$

where

Y_t = height of water table above tile on day t,

t = time in days,

f = drainable porosity,

P = precipitation on day, t, in depth,

j = reservoir coefficient in days,

n = consecutive odd integers, and

exp = the base of the natural logarithm raised to an exponent.

The reservoir coefficient, j, was defined as:

$$j = (fS^2)/(\Pi^2 KD)$$

where

S = drain spacing (See Figure 2),

K = hydraulic conductivity, and

D = average depth of flow above the impermeable layer.

Expanding Equation 1 for n = 2,

$$Y_t = [(4P_t)/(f\Pi)]\ (j)\ [1-\exp(-t/j)] - (1/27)\ [1-\exp(-9t/j)] + r_2 \qquad 2a$$

where r was defined as the remainder after n terms of the infinite series. By using the designation of Kraijenhoff, Equation 2a may be written as

$$Y_t = Y_t^* - Y_t^{**} + r_2. \qquad 2b$$

The above equations were used to calculate the behavior of the water table midway between the outflow drains. It was assumed that a given time distribution of percolation could be approximated by a succession of intervals with constant percolation rates. This was supported by work previously reported by Childs (10).

In a later paper, Kraijenhoff (35) discussed a scheme for prorating the affect of the one day's water table movement on that of successive days. When a time interval of one day was substituted in Equation 2a, the following results were obtained:

$$Y_1 = [(4P_1)/(f\Pi)] \ (j) \ [1-\exp(-1/j)] \\ - (1/27) \ [1-\exp(-9/j)] + r_{2_1}. \quad 3$$

The height of the water table at the end of the second day of precipitation was the sum of the components derived from the first and second days. As soon as the second interval (day) started, the rate of precipitation changed from P_1 to P_2. Let

$$Y_2 = Y_{d_2} + Y_{P_2} \quad 4$$

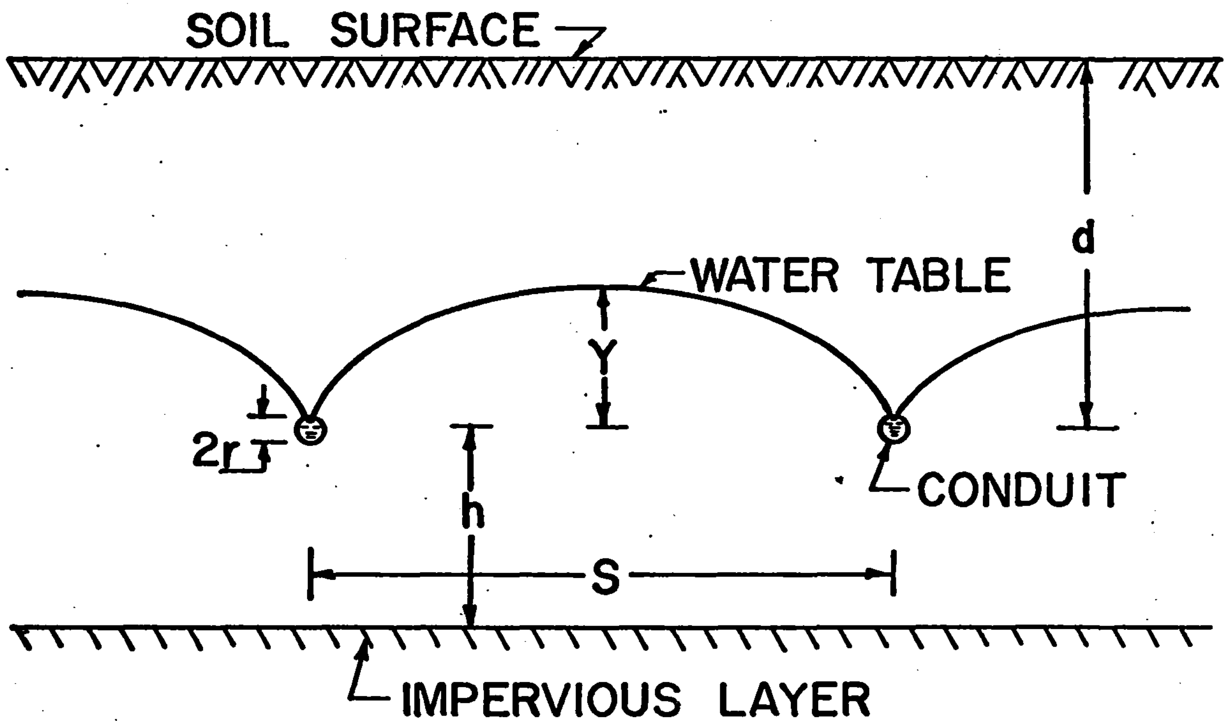
where

Y_{d_2} = the change in depth of water table at the end of the second day where the previous day had a precipitation rate of P_1, and

Y_{P_2} = the change in depth of water table at the end of the day which had a precipitation rate of P_2.

The change in depth Y_{d_2} was computed as the water table depth that would be caused by the precipitation rate of P_1 over one day.

Figure 2. General profile of water table with tile drain installation


SOIL SURFACE
d
WATER TABLE
Y
2r
CONDUIT
h
S
IMPERVIOUS LAYER

$$Y_{d_2} = [(4P_1)/(f\Pi)]\,(j)\,[1-\exp(-2/j)]$$

$$-\ (1/27)\ [1-\exp(-18/j)]\ +r_a$$

$$-\ [(4P_1)/(f\Pi)]\ (j)\ [1-\exp(-1/j)]$$

$$-\ (1/27)\ [1-\exp(-9/j)]\ +r_{2_1} \qquad \text{5a}$$

Since r_a was nearly equal to r_{2_1}, these terms were deleted, and Equation 5a was reduced to:

$$Y_{d_2} = [(4P_1)/f\Pi)]\ (j)\ [1-\exp(-1/j)][\exp(-1/j)]$$

$$-(1/27)\ [1-\exp(-9/j)][\exp(-9/j)] \qquad \text{5b}$$

Again, using the notation of Kraijenhoff, Equation 5b was written as

$$Y_{d_2} = Y_1^{*}[\exp(-1/j)]\ -Y_1^{**}[\exp(-9/j)] \qquad \text{5c}$$

The second component of Y_2 was determined by calculating the change in water table due to the precipitation rate of P_2 over the second day as

$$Y_{P_2} = [(4P_2)/(f\Pi)]\ (j)\ [1-\exp(-1/j)]$$

$$-\ (1/27)\ [1-\exp(-9/j)]\ +r_{2_2}. \qquad 6$$

Adding the components,

$$Y_2 = Y_1^{*}\ [\exp(-1/j)] + [(4P_2)/(f\Pi)]\ (j)\ [1-\exp(-1/j)]$$

$$-Y_1^{**}[\exp(-9/j)]$$

$$- [(4P_2)/(f\Pi)]\ (j/27)\ [1-\exp(-9/j)]\ +r_{2_2}. \qquad 7$$

Likewise, Y_3 was obtained by letting

$$Y_3 = Y_{d_3} + Y_{P_3}$$

where

$$Y_{d_3} = Y_2^{*}\ [\exp(-1/j)] - Y_2^{**}\ [\exp(-9/j)],$$

and

$$Y_{P_3} = [(4P_3)/(f\Pi)]\ (j)\ [1-\exp(-1/j)]$$

$$- (1/27)\ [1-\exp(-9/j)]\ +r_{2_3}.$$

Then

$$Y_3 = Y_2^{*}\ [\exp(-1/j)] + [(4P_3)/(f\Pi)]\ (j)\ [1-\exp(-1/j)]$$

$$-Y_2^{**}[\exp(-9/j)] - [(4P_3)/(f\Pi)]\ (j/27)\ [1-\exp(-9/j)]$$

$$+r_{2_3}.$$

By use of the same reasoning, the general form of Equation 7 was written as

$$Y_t = Y_{t-1}^{*}[\exp(-1/j)] + [(4P_t)/(f\Pi)]\ (j)\ [1-\exp(-1/j)]$$

$$-Y_{t-1}^{**}[\exp(-9/j)] - [(4P_t)/(f\Pi)]\ (j/27)\ [1-\exp(-9/j)]$$

$$+r_{2_t}. \qquad 8$$

Kraijenhoff defined a portion of the hydrograph as a recession curve. For practical purposes it was assumed that the recession curve had started as soon as the second term in the series of

Equation 1 became smaller than 1.0 percent of the first term. It was reported that if the logarithm of the water table depths, Y, was plotted against the corresponding values of time, a linear relationship resulted which produced the equation

$$\tan \alpha = \frac{1}{2.303j}, \qquad 9$$

where α = the angle of inclination which the recession curve made with the axis on which time was plotted (See Figure 9). In an actual drainage situation the reservoir coefficient, j, could be determined if the recession curve was sufficiently approximated.

Kraijenhoff (34) also studied the symmetrical free-surface flow of groundwater to outflow channels in a scaled granular model. The study was directed toward the affect of the unsaturated zone on nonsteady conditions for open channels, and there was some discrepancy between the water table depths observed in the model and those calculated by the analytical procedure.

Van Schilfgaarde (70) used a procedure similar to Kraijenhoff's transient flow analysis, except that van Schilfgaarde's equation was derived from a steady-state solution based on potential theory while Kraijenhoff's development was derived from a steady-state solution based on the Dupuit-Forchheimer assumptions. Van Schilfgaarde used the solution given by Kirkham (27) which may be stated as follows:

$$Y = (SP/K)(1/\Pi)\left\{\ln\ (S/\Pi r) + \sum_{m=1}^{\infty} (1/m)\ [\cos(2m\Pi r/S) - \cos m\Pi]\ [\coth\ (2m\Pi h/S)\ -1]\right\} \qquad 10$$

where

Y = height of water table above the level of the tile axes,

S = drain spacing,

P = average rate of percolation,

K = hydraulic conductivity,

r = drain radius,

h = distance below the drain to an impervious layer, and

m = consecutive integers.

If $F(r/S,\ h/S)$ can be defined by the expression

$$F = (1/\Pi)[\ln(S/\Pi r) + \sum_{m=1}^{\infty} (1/m)]\ [\cos(2m\Pi r/S) - \cos m\Pi]\ [\coth(2m\Pi h/S)-1], \qquad 11$$

then

$$Y = SPF/K. \qquad 12$$

Following the technique of Bouwer and van Schilfgaarde (7) an expression involving P,

$$dY/dt = -\ P/(Cf), \qquad 13$$

was established where C was defined as the ratio of the average flux between the drains to the flux midway between the drains, and P was the instantaneous drainage rate. The instantaneous drainage rate was taken equal to the percolation rate on the assumption that midway between drains the instantaneous drainage rate was the same as the steady-state drainage rate for the same height of water table. The following equation was obtained by substituting in Equation 12:

$$-fC(dY/dt) = (YK)/(SF). \qquad 14$$

By substituting the factor A for the relationship fFCS/K, Equation 14 was written as

$$dY/dt + Y/A = 0. \qquad 15$$

Van Schilfgaarde solved this equation by application of the Laplace transform method. Once the proper function was selected, it was applied to Equation 15, as a forcing function which represented an impulse. The intermediate steps pertaining to the Laplace transform are given in Appendix B. By using the unit step function, van Schilfgaarde was able to obtain an expression for the pulse of the water table at some time, t, as

$$Y(t) = Y_o e^{-(t-t_i)/A} U(t-t_i) \qquad 16$$

where

Y_o = magnitude of pulse or initial value of Y,

t = duration of the pulse,

t_i = starting time, and

$U(t-t_i)$ = unit step function.

Continuous precipitation p(t) over the time interval $t_o<t<b$ was divided into n equal subdivisions such that $t_o = T_1$, $t_1 = T_2$, $t_2 = T_3$, ..., $b = T_{n+1}$,

and

$$\Delta T = T_{j+1} - T_j \tag{17}$$

where

$j = 1, 2, \ldots n.$

It was assumed that the rate $p(t)$ remained constant and equal to $p(T_j)$ over the increment T_j, T_{j+1}.

Then, with the pulse height

$$Y_o = p(T_j)\Delta T/f,$$

the response for each interval was found by substituting in Equation 16 to obtain

$$Y_j = p(T_j)(\Delta T/f)\ e^{-(t-t_i)/A} \tag{18}$$

where

f = drainable pore space.

Note that $U(t-t_i)$ was deleted, since by the nature of the unit step function if $t>T$, the step function takes the value of unity.

The total response for precipitation $p(T)$ during $t_o<T<b$ was determined by using the principal of superposition. Briefly stated, superposition (48, p. 278) has been defined as:

> ...The sum of a linear system to a number of simultaneously applied excitations is equal to the sum of the responses to the excitation, taken one at a time.

By adding the water table heights due to each pulse of precipitation, the following equation was obtained:

$$Y(t) = \lim_{\substack{n\to\infty \\ \Delta T\to o}} \sum_{j=1}^{n} Y_j. \qquad 19a$$

Substituting:

$$Y(t) = \lim_{\substack{n\to\infty \\ \Delta T\to o}} \sum_{j=1}^{n} e^{-(t-t_j)/A} p(T_j)\,\Delta T/f, \qquad 19b$$

or

$$Y(t) = 1/f \int_{t_o}^{b} p(T)\ e^{-(t-T)/A} dT. \qquad 19c$$

Equation 19c was a general form of a solution for a continuous precipitation. For the special case where p(T) remains constant and equal to P over the increment (T_j, T_{j+1}), Equation 19c can be integrated, resulting in

$$Y(t) = (AP/f)\,[\,e^{-(t-b)} - e^{-(t-t_o)/A}]. \qquad 20$$

To apply Equation 20 to a designated percolation sequence, van Schilfgaarde considered time intervals of equal length where P_N represented the percolation rate during the N^{th} time period. Then, by superposition, the water table at the end of the N^{th} time period was

$$\begin{aligned} Y_N = A/f \Big\{ & P_1\,[\,e^{-(N-1)/A} - e^{-(N/A)}] \\ & + P_2\,[\,e^{-(N-2)/A} - e^{-(N-1)/A}] \\ & \cdot \\ & \cdot \\ & \cdot \\ & + P_N\,[\,e^{-(N-N)/A} - e^{-(N-N+1)/A}] \Big\}. \end{aligned} \qquad 21$$

The interval of time was conveniently taken as one day. The value of P was determined from a water balance which required a root zone at field capacity before P could have a moisture content greater than zero.

Water Balance

The water-balance approach has been used by several investigators for the purpose of gaining information on the frequency of certain soil moisture levels. The pertinent variables generally included precipitation, runoff, water-holding capacity of the soil, evaporation, and transpiration. Runoff and evapotranspiration are usually the more difficult factors to determine.

Thornthwaite (62) introduced the concept of potential evapotranspiration and its relation to the climate of a region. The procedure involved the determination of the monthly evapotranspiration as follows:

$$e = ct^a \qquad (22)$$

where

e = monthly evapotranspiration in units of length,

t = mean monthly temperature in degrees centigrade, and

c and a were constants dependent upon climate.

Adjustments were necessary to compensate for number of hours of sunshine per day and the number of days per month. Precipitation, storage, actual evaporation, and runoff were the main factors in Thornthwaite's water balance. Water surplus and water deficiency could also be determined.

Penman (52) used a combination of energy balance and aerodynamic theories to determine evaporation from different surfaces. It was stated that evaporation from an open water surface was highly correlated to potential evapotranspiration. The four climatological parameters involved in this procedure included air temperature, wind velocity, relative humidity, and sunshine duration.

Veihmeyer and Hendrickson (72) proposed that evapotranspiration continued at the potential rate until all of the available water had been used from the root zone. Marlatt et al. (47) found good agreement between calculated and observed amounts of soil moisture when the ratio of actual to potential evapotranspiration was constant at 1.0 during the depletion of the first inch of soil moisture below field capacity, The agreement continued beyond that when it was assumed that the ratio decreased linearly to zero at the permanent wilting percentage. This trend was further supported by Denmead and Shaw (13) in studies of transpiration from corn under field conditions in Iowa. They observed that actual transpiration took place at the potential rate for some time as the soil was depleted below field capacity. On a day when the potential for transpiration as determined by meteorological conditions was low, actual transpiration occurred at this potential rate even though soil moisture was near the permanent wilting point. Also, on a day when the potential for transpiration was very high the actual rate dropped below the potential rate when the soil moisture was only slightly below field capacity.

Van Bavel (65) was one of the early investigators to make use of the water balance in estimating drought hazard. Studies were conducted for several areas, an example being that for the Lower Mississippi Valley (64). In that study the moisture balance was used to predict the occurrence of water surplus as well as drought. The analysis was run for several values of available-moisture capacity at various stations. Each day an estimated value for evapotranspiration was subtracted from soil moisture storage, and the precipitation for that day added. If the amount of moisture exceeded the available moisture-holding capacity, the excess was considered runoff. Otherwise, it was assumed that all precipitation went into soil storage. A similar investigation was made by Blake *et al.* (5) for the state of Minnesota.

Wiser and van Schilfgaarde (76) assumed that the soil does not wet above a given moisture content, which was considered to be field capacity, and that the water supplied by precipitation moved down into the soil only when all of the soil above it was at field capacity. Any precipitation beyond that sufficient to satisfy the total soil moisture capacity was designated excess. Four different soil moisture capacities were used.

Stol (59) described the climate aspect of a water balance with a statistical analysis of the difference between precipitation and evaporation. Use was made of the 10-day totals of measured precipitation (P) and the evaporation (E_o). The difference between precipitation and evaporation was calculated by the expression ($P-XE_o$), in which X was the reduction factor for E_o. Results using this formula were evaluated for periods of 10, 20, 30, 60, 90, 120, and 180 consecutive days respectively calculated from the first of each month. The factor, X, was varied from 0.1 to 1.0. By starting at some known soil moisture content, a balance was determined such that moisture deficiency was designated as evaporation surplus and moisture excess was designated as precipitation surplus. Each period was expressed in terms of a summated relative frequency. Then by integrating the frequency curves for the various periods into a collection of frequency polygons, it was possible to obtain the surplus probability for either rainfall or evaporation.

Ligon *et al*. (38) presented an approach to the water balance which involved different relationships between actual evapotranspiration and the potential evapotranspiration. The former was equal to the latter when the daily precipitation was less than 0.01 inch and the moisture content of the soil near the surface was above the wilting point. The soil profile was divided into two zones such that the upper zone was that portion of the total root zone which held 1.0 inch of the available moisture. When there was no available moisture in the upper horizon, the estimated evapotranspiration equalled potential evapotranspiration times the ratio of the amount of available moisture carried over in the lower horizon to the total amount of available moisture which the lower horizon was capable of holding. The daily moisture change was determined by balancing precipitation against evapotranspiration. It was assumed that precipitation first brought the surface layer to field capacity, with continued percolation moving downward into the lower layers. Finally, when the field capacity of the lower layers was exceeded, the remainder was considered excess, either surface runoff, or deep percolation. No provisions were made to determine the amount of runoff which could have occurred when the moisture content of the soil profile was below field capacity.

Shaw (57) developed an empirical method to estimate soil moisture under corn in Iowa. The procedure involved the water balance within the limits of the soil profile's permanent wilting point and field capacity. Results from 10 years of soil moisture sampling were used to develop a procedure for the purpose of determining moisture deficiencies. Soil moisture-holding capacities of 6, 9, and 12 inches, and initial moisture profiles of 20, 60, and 100 percent of field capacity were used for soil profiles of 5 feet. Thirty years of rainfall records were used to compute a water balance for each of the 9 moisture combinations. Then by selecting the proper moisture-holding capacity and the appropriate initial soil moisture profile, it was possible to follow the moisture balance for a particular soil condition. The accuracy of the predicted soil moisture improved after the first runoff had occurred since this tended to offset any error which may have developed due to the chance of not starting with the correct initial soil moisture.

Buss and Shaw (8) used the method of Kohler and Linsley (33) to estimate runoff which was used in Shaw's water balance. Kohler and Linsley selected five variables to be used in an analysis for runoff correlation. These variables included basin recharge, antecedent-precipitation index, season or weeks of the year, storm duration and storm rainfall. The correlation procedure permitted the establishment of the relative relationship among the variables in graphical form, such that by entering with a known antecedent precipitation index, the corresponding storm runoff in inches could be obtained. It was pointed out that certain deficiencies prevailed which should not be overlooked. Rainfall intensity was omitted and frozen conditions hindered the procedure. The omission of intensity was partly compensated for by reducing a long storm into several short periods of rainfall, considering all rainfall occurring prior to any specific period as antecedent precipitation. It was further pointed out that neglecting intensity apparently caused serious error in total storm runoff only when intensities were so great that infiltration capacities were exceeded. It was suggested that fair results were obtained during frozen conditions when the weekly curve representing maximum runoff conditions was used regardless of the date of the storm. Kohler and Linsley did not obtain good agreement for storms which were predominatly snow, but found that when only a light snow cover remained at the end of a storm, the estimated water equivalent could be subtracted from the observed storm precipitation. Snow on the ground at the beginning of the storm was included in the storm precipitation, rather than in the antecedent precipitation, if it was dissipated during the storm.

In making use of the Kohler and Linsley procedure Shaw (57) used the late June period (22nd week of the year), and a duration of zero hours. The antecedent precipitation index was given as

$$API = P_1/t_1 + P_2/t_2 + \ldots + P_i/t_i \qquad 23$$

where

P_i = the amount of precipitation that occurred i days prior to the day being considered,

and

t_i = the corresponding number of days.

It was found that when the top 3 feet or more of the root zone were at field capacity, a correction for amount of runoff was needed. To avoid overcorrection for small rains, and to allow for greater runoff from heavy rains, the precipitation index was modified such that for all rains of 1.0 inch or greater, half of the precipitation amount was added to the index for the day having more than 1 inch of rainfall. The revised index was

$$API = P_1/t_1 + P_2/t_2 + \ldots + P_i/t_i + P_o/2 \qquad 24$$

where

P_o = the precipitation amount for which runoff was being computed.

P_o was zero when the precipitation was less than 1.0 inch. On subsequent days, $P_o/2$ was carried in the expression as P_i.

Equation 24 was used to predict runoff in the spring when the ground was either bare, or sparsely covered, and during the summer when high intensity rains were expected to occur. It was assumed that a good crop cover existed during the fall, therefore Equation 23 was used to compute runoff during that season.

Drainable Porosity - Hydraulic Conductivity Relationship

Linsley (42) defined the volume of water free to drain from the acquifer as the specific yield. This nomenclature was used by the Bureau of Reclamation (18). Taylor (61) used drainable porosity to define the volume fraction of pore water which could be drained from a soil under prescribed conditions.

Baver (3) observed that soils with apparent low hydraulic conductivities always had a small content of non-capillary pores. Muskat (49) indicated that permeability was dependent upon the dimensions of the pore. It is well known that when non-capillary porosity is being determined, the tension at which the soil is drained must be considered since the tension has an effect upon the amount of non-capillary pores.

Klinge (31) did not find a good relationship between the percent volume of large pores and hydraulic conductivity in a silt-loam soil. Peele (51) found the relationship of percolation rate under unit head to the volume of pores drained in 30 minutes under 60 cm of tension to be

$$\log_{10}K = 1.489 \log_{10}f - 0.70874,$$

where

K = percolation rate in inches per hour, and

f = volume of pores drained in 30 minutes in percent of pore volume.

This relationship was obtained from a large sample of soils from the Southeastern part of the United States.

Mason *et al.* (48) measured hydraulic conductivity and percentage of pores drained at 60 cm tension for 15 hours for about 8,000 individual core samples from approximately 900 sites in 7 states. As expected, it was found that a decrease in hydraulic conductivity, percentage of pores drained, and bulk density were associated with an increase in the amount of silt and clay. The hydraulic conductivity was more related to percentage of large pores than to the bulk density.

Dumm[2] confirmed the relationship used by the Bureau of Reclamation as

$$f = 0.1151 \log_{10} K + 0.1005$$

where

f = specific yield (drainable porosity) in a fraction form for a range between 0.05 and 0.35, and

K = hydraulic conductivity in inches per hour.

This equation was developed from a general relationship curve for hydraulic conductivity and drainable porosity measurements made by the Bureau of Reclamation, and from measurements of hydraulic conductivity and "large drainable pore space" made by the Soil Conservation Service from throughout the United States. Dumm further related that the samples

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[2]Dumm, L. D. Denver, Colorado. Data relating specific yield to hydraulic conductivity. Private communications. 1965.

used for the relationship between K and f were from a depth deeper than the A-horizon since it was felt that conditions near the drains gave a better representation since this composed the region where the maximum water movement occurred.

INVESTIGATIONS

The final goal of the overall investigation was to be able to describe the degree and frequency of excess soil-moisture conditions caused by precipitation. First, it was necessary to separate the total precipitation into those proportions which enter the soil, leave as surface runoff, or evaporate or transpire to the atmosphere. This was somewhat analogous to defining the hydrologic cycle; a schematic illustration is given in Figure 3. Although it was recognized that the moisture cycle is much more complex than indicated, it serves to show how the total supply of moisture was divided into general categories.

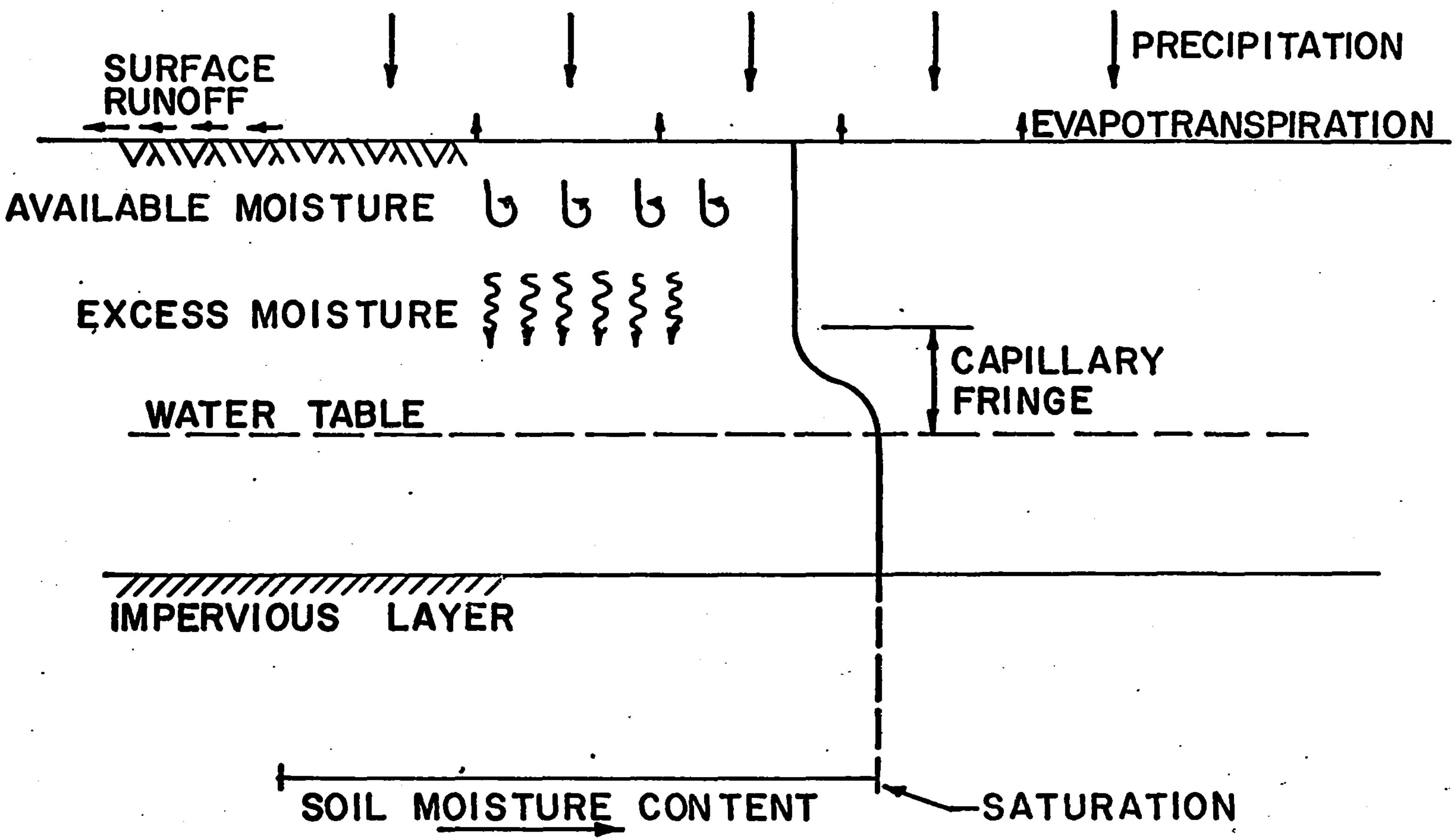
The assumption was made that the water which infiltrated into the soil moved down only when all of the soil above was at field capacity. The capillary pore space of the soil forms a reservoir capable of holding moisture which is available for plant use (see Figure 3). This is commonly referred to as the available water-holding capacity (AWC) of the soil. When the entire root zone has reached field capacity, the moisture content of the soil may be indicated as shown by the upper portion of the moisture-content curve in Figure 3.

The portion of the total rainfall which creates undesirable soil-moisture conditions was of major importance. The non-capillary pores play a significant role in that they also compose a storage facility, but one which is undesirable for the development of microorganisms and plant roots. As these pores store the excess water which is moving into the soil, a water table is formed, shown in Figure 3, which continues to fluctuate under normal field conditions. Artificial drainage is capable of removing this water since it moves under the influence of gravity, thereby causing the water table to fluctuate. Hence, the water table was used as a criterion in determining the degree of adverse soil-moisture conditions. Certain predictions could be made concerning the frequency of a given water table level based on recorded water table levels of the past. Since only short term records of the water table were available, it was necessary to develop procedures by which the water table fluctuations could be predicted or calculated.

Moisture Storage Relations

The moisture-storage capacity of a soil is a significant factor, and one often neglected, in determining the amount of water that must be drained from a tile drainage system for a given precipitation pattern. This can be illustrated by considering the results of a one-inch rain which occurred uniformly with a duration which permitted the entire amount to infiltrate through the surface of the soil. Assume that the soil profile was such that each foot, for a depth of five feet, held two inches of water between the wilting point and

Figure 3. Schematic diagram of water balance illustrating how excess moisture builds a water table


PRECIPITATION
SURFACE RUNOFF
EVAPOTRANSPIRATION
AVAILABLE MOISTURE
EXCESS MOISTURE
CAPILLARY FRINGE
WATER TABLE
IMPERVIOUS LAYER
SOIL MOISTURE CONTENT
SATURATION

field capacity, and also assume that the soil had a drainable porosity (non-capillary pore space) of 10 percent. Consider the case when the first foot increment of the soil profile was at 50 percent of the AWC, the second foot at 80 percent, and the rest of the profile at 100 percent of the AWC. Then the storage capacity for available water was 1.0 inch in the first foot and 0.4 inch in the second foot. In this case, the 1.0 inch of precipitation was readily stored as available water in the first foot, though it would be suspected that some moisture moved into the second foot to satisfy equilibrium conditions imposed by capillary forces. Now, consider the case when the first foot was at 80 percent of AWC and the remainder of the 5-foot profile was at field capacity. Then only 0.4 inch of the 1.0 inch rainfall was stored as available water. However, the remaining 0.6 inch was stored in the non-capillary pore space, and due to the force of gravity, percolated to the lower level of the root zone where a free water surface was initiated. This water level eventually stood at a height of 0.5 foot above the impervious layer.

In practice, there are varying amounts of deep percolation. The reversal of deep percolation takes place where artesian flow is encountered. In this investigation, it was assumed that an impervious layer existed (1) at a depth of four feet below the tile drains, or (2) at a depth of 0.5 foot below the drains. In all cases, the depth of tile was assumed to be four feet below the soil surface. The impervious layer restricted deep percolation, and all non-capillary storage above the tile lines was drained artificially.

Excess Moisture Determined from Water Balance

In accordance with the above discussion, the problem was limited to investigation of the transient water table behavior where flow of water to drains through the unsaturated zone was excluded. A daily water balance based on the procedures of Shaw (57) was used to evaluate the time and amount of excess moisture. Daily precipitation data, as reported by the Ames, Iowa station, were used to determine excess moisture for the years 1933-1962. This method was developed specifically for continuous corn with a root zone of five feet. It was also necessary to know the AWC of the root zone and the initial amount of moisture in the profile at the beginning of April each year. The water balance was carried out only for the time interval April 1 to November 30. The moisture status of the soil from December 1 to March 31 was estimated by an equation developed by Shaw (58) which was based on a regression analysis dependent upon the precipitation over this period. Runoff was estimated as previously explained. Evapotranspiration (ET) was determined differently for the periods: (a) April 1 through June 6, (b) June 7 through

September 30, and (c) October 1 through November 30. Evaporation (E) was the major source of water loss during the first period. Since moisture loss occurred primarily from the top six inches of soil during this period, a good approximation could be obtained by using an average evapotranspiration rate of 0.1 inch per day.

The end of the first week in June was used as the breaking point between the first and second periods because of a distinct change in the ratio of evapotranspiration to open-pan evaporation at that time. The open-pan evaporation was the basic factor used for estimating ET during this period. The pan evaporation was adjusted by a factor which was dependent upon the stage of crop development, and another factor related to the moisture stress conditions. Denmead and Shaw (14, 13) developed criteria for evaluating these factors.

After October 1, evapotranspiration was assumed to be 35 percent of pan evaporation. This factor was estimated from data pertaining to the relationship between stage of plant growth and open-pan evaporation. After November 1, when evaporation-pan data were not available, evaporation was assumed to be 0.1 inch per week.

Water was assumed to be removed from each foot-increment in the pattern shown in Table 1.

Table 1. Water extraction from the soil profile at different depths during the growing season [After Shaw (57, p. 974)]

Date	ET or E which came from respective depths %	Depths from which water was extracted Ft.
4-1 to 5-7	100	Upper half of 1st
5-8 to 5-14	100	1st
5-15 to 5-27	67.7, 33.3	1st, 2nd
5-28 to 6-4	60, 20, 20	1st, 2nd and upper half of 3rd
6-5 to 6-11	60, 20, 20	1st, 2nd, 3rd
6-12 to 6-18	60, 15, 15, 10	1st, 2nd, 3rd and upper half of 4th
6-19 to 6-25	60, 15, 15, 10	1st, 2nd, 3rd, and 4th
6-26 to 7-1	60, 10, 10, 10[a]	1st, 2nd, 3rd, 4th and upper half of 5th
	60, 15, 15, 10[b]	1st, 2nd, 3rd and 4th
after 7-2	60, 10, 10, 10[a]	1st, 2nd, 3rd, 4th and 5th
	60, 15, 15, 10[b]	1st, 2nd, 3rd and 4th

[a]Used only if first 4 feet all have less than 50 percent available moisture.

[b]Used if any of first 4 feet have greater than 50 percent available moisture; however, after August 1, the percent available is always computed on the total available water in the 5-foot profile.

When an increment of soil other than the top foot had no available water, the amount was prorated among the other depths from which water was being extracted. When the top foot had no available water, the extraction pattern was shifted one depth deeper. The amount normally extracted from the greatest active depth was divided equally among the other active depths. As long as available water was present in the upper six inches after June 7, an additional 0.1 inch was added to the effective evapotranspiration to compensate for the higher evaporation from the soil surface.

The above empirical method for evaluating the soil moisture status was programmed on an IBM 7074 computer (12). The soil moisture profile for the beginning of the first year was estimated to be at 100 percent of the AWC on the basis of the amount and distribution of precipitation for the previous year. The precipitation for 1932 was 2.96 inches above normal with the above normal conditions occurring in August and November of that year.

The basic balance was expressed as

$$SW_t = SW_{t-1} + PCP_t - RNF_t - ET_t \qquad 22$$

where

SW_t = soil moisture content at the end of day t,

SW_{t-1} = soil moisture content at the end of the previous day,

PCP_t = precipitation on day t,

RNF_t = runoff on day t and

ET_t = evapotranspiration on day t.

An abbreviated example of the output from the computer is given in Table 2. The column headings are explained as follows:

MO: Month, 4 for April, ... 11 for November.

Table 2. Sample output of water balance from IBM 7074 computer for 9-inch available water-holding capacity under corn in central Iowa, June, 1947

MO	DY	PCP	EVP	ET	1	2	3	4	5	6	7	8	9	10	RNF	STET	TOTAL	EXCS
6	1	2.75	0.18	0	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	1.06	0.00	9.00	1.36
6	2	0.01	0.20	0	1.01	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.00	8.91	0.00
6	3	0.00	0.27	0	0.91	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.00	8.81	0.00
6	4	0.60	0.00	0	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.00	9.00	0.31
6	5	0.40	0.24	0	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.00	9.00	0.30
6	6	0.00	0.30	0	1.00	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.00	8.90	0.00
6	7	0.10	0.11	4	1.08	1.08	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.04	8.96	0.00
6	8	0.00	0.23	9	1.03	1.03	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.09	8.86	0.00
6	9	0.00	0.17	7	1.00	1.00	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.07	8.79	0.00
6	10	0.10	0.32	13	1.03	0.93	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.13	8.76	0.00
6	11	0.00	0.21	9	0.99	0.89	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.09	8.67	0.00
6	12	3.17	0.04	2	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	1.34	0.02	9.00	1.49
6	13	1.22	0.13	6	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.62	0.06	9.00	0.54
6	14	0.00	0.15	6	1.08	1.08	0.84	0.84	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.06	8.94	0.00
6	15	0.00	0.33	15	1.03	1.03	0.82	0.82	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.15	8.79	0.00
6	16	0.00	0.37	16	0.98	0.98	0.79	0.79	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.16	8.63	0.00
6	17	0.59	0.10	5	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.05	9.00	0.17
6	18	0.18	0.11	5	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.05	9.00	0.13
6	19	0.00	0.08	4	1.09	1.09	0.84	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.04	8.96	0.00
6	20	0.18	0.14	7	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.07	9.00	0.08
6	21	1.06	0.07	3	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.18	0.03	9.00	0.85
6	22	0.00	0.05	3	1.09	1.09	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.03	8.98	0.00
6	23	2.30	0.22	11	1.10	1.10	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.76	0.11	9.00	1.40
6	24	0.00	0.25	13	1.06	1.06	0.83	0.83	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.13	8.87	0.00
6	25	0.12	0.24	13	1.10	1.05	0.81	0.81	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.13	8.86	0.00
6	26	0.00	0.26	14	1.05	1.00	0.78	0.78	0.85	0.85	0.85	0.85	0.85	0.85	0.00	0.14	8.72	0.00
6	27	0.00	0.36	20	0.99	0.94	0.76	0.76	0.81	0.85	0.85	0.85	0.85	0.85	0.00	0.20	8.53	0.00
6	28	0.00	0.23	13	0.96	0.90	0.75	0.75	0.78	0.85	0.85	0.85	0.85	0.85	0.00	0.13	8.40	0.00
6	29	0.21	0.33	19	1.10	0.86	0.73	0.73	0.75	0.85	0.85	0.85	0.85	0.85	0.00	0.19	8.42	0.00
6	30	0.43	0.24	14	1.10	1.10	0.82	0.72	0.72	0.85	0.85	0.85	0.85	0.85	0.00	0.14	8.71	0.00

DY: Day number within month.

PCP: 24-hour precipitation, specifically for Ames, from 7 p.m. of one day to 7 p.m. of the following.

EVP: 24-hour evaporation in inches from a Weather Bureau Standard Class A evaporation pan for Ames, the 24 hours ending at 7 p.m.

ET: 24-hour potential evapotranspiration from corn in hundredths inch as estimated by applying stage of crop factor to EVP.

1: Inches of available soil moisture in the top half-foot of soil profile.

2: Inches of available soil moisture from 0.5 to 1.0 foot depth.

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10: Inches available soil moisture from 4.5-to 5.0-foot depth.

RNF: Estimated runoff in inches using antecedent precipitation index (API). In the computer program the daily precipitation series used to estimate the API was arbitrarily started 4 days before the first day of budget.

STET: ET reduced by the moisture stress factor.

TOTAL: Total available soil moisture in inches in top 5 feet of profile.

EXCS: Excess as determined by precipitation, available storage, runoff, and evapotranspiration.

An equation for determining excess (EXCS) was developed as

$$EXCS_t = PCP_t - (TOT_t - TOT_{t-1}) - RNF_t - ET_t \qquad 23$$

where

t = day on which excess was determined,

PCP = precipitation,

TOT = total available soil moisture in entire root zone,

RNF = runoff, and

ET = evapotranspiration.

Soil conditions where the AWC was 9.0 inches were selected to represent the average soil moisture conditions in central Iowa. It was recognized that a more complete analysis should have included possibly a 6.0-inch and a 12.0-inch AWC soil. The assumed AWC was distributed within the 5-foot profile in accordance with general findings of the Iowa Soil Moisture Survey (11). For 100 percent AWC, this amounted to 1.10 inches for each of the first 2 half-foot increments, and 0.85 for the next 8 half-foot increments. Dale (11) stated that results of the moisture survey indicated that when only 60 percent of the available water-holding capacity existed, the distribution of moisture in early spring for a 9-inch AWC soil could be established as 1.0 inch for each of the first 2 half-foot increments, 0.59 inch each for the third and fourth half-foot increments, 0.30 inch for the fifth through eighth half-foot increments, and 0.51 inch for the ninth and tenth half-foot increments. This general distribution scheme was built into the computer program in such a manner that when the AWC was between 60 and 100 percent in the spring, the distribution as presented for 100 percent AWC was used; when the AWC was below 60 percent, the proportional distribution as given for the 60 percent AWC was used. The proportional distributions were obtained by using the 100 percent AWC as a base. For example, at 60 percent AWC or lower, the amount of moisture in the first half-foot increment was obtained by multiplying the total initial moisture in the 5-foot profile by the ratio 1.0/5.4, or 0.186, where 1.0 corresponded to the inches of moisture in the respective increment when the entire profile contained 5.4 inches (60 percent AWC). The ratio for the first increment when the AWC was 100 percent, amounted to 1.1/9.0, or 0.123. The ratio for other increments was obtained in a like fashion. The respective soil moisture contents for the last of November, winter addition, and initial profile in the spring are given in Table 3. The additional moisture added over the winter period was estimated by the procedure developed by Shaw (58). The initial profile was obtained by adding the previous values for the November ending and winter addition, with the restriction that the maximum initial profile could be only nine inches.

Table 3. Initial profile, end of November, and winter-addition moisture amounts for a 5-foot soil profile with a 9-inch AWC, from 1933 to 1962, Ames, Iowa

Year	Initial profile	End of Nov.	Winter add.	Year	Initial profile	End of Nov.	Winter add.
1933	9.00	4.51	0.82	1948	9.00	6.64	4.19
1934	5.33	7.86	3.87	1949	9.00	5.53	1.32
1935	9.00	8.96	2.71	1950	6.85	2.01	4.79
1936	9.00	5.19	3.80	1951	6.80	8.90	4.02
1937	9.00	2.39	4.27	1952	9.00	4.35	4.90
1938	6.66	3.69	2.59	1953	9.00	2.65	3.05
1939	6.28	2.23	2.79	1954	5.70	8.54	2.46
1940	5.02	6.59	3.07	1955	9.00	2.05	0.85
1941	9.00	8.78	3.40	1956	2.90	4.12	3.35
1942	9.00	7.70	3.30	1957	7.47	8.05	2.27
1943	9.00	8.11	4.13	1958	9.00	6.94	4.43
1944	9.00	6.39	5.00	1959	9.00	7.94	5.59
1945	9.00	5.49	4.22	1960	9.00	7.02	3.75
1946	9.00	7.93	4.13	1961	9.00	8.84	2.78
1947	9.00	7.48	4.39	1962	9.00	4.28	4.69

Viscous Fluid Model

The advantage offered by dimensional analysis was considered worthy of pursuing to see whether a dimensionless plot could be developed consisting of depth to water table, time, hydraulic conductivity, drainable porosity, and depth to impervious layer below tile line. The depth of tile was considered to be constant. Curves developed by this method would have made it possible to determine the duration of a given water table depth for a known hydraulic conductivity, drainable porosity, and depth to the impervious layer.

Scales

Ligon et al. (39) investigated the application of similitude to the modeling of unsteady-state soil drainage problems, particularly the problem of the falling water table between open ditch drains. Glass spheres approximately 2 mm in diameter were used as the porous medium and glycerol was used as the model fluid. It was found that three simplifying assumptions were valid, namely; (a) that the effect of a capillary fringe in the model could be eliminated, (b) that all the effects of fluid characteristics, acceleration of gravity, and characteristics of the porous material could be taken into account by the hydraulic conductivity, K, of the system, and the drainable porosity, f, of the medium, and (c) that flow occurred in two-dimensional planes perpendicular to the drains.

The following variables were considered pertinent for a viscous-fluid model used to study water-table behavior:

Z , drawdown of the water table midway between tiles (See Figure 2) (L)

t, time measured from the beginning of the period being studied (T)

S, drain spacing (L)

d, drain depth (L)

h, height of tile above an impervious layer (L)

K, hydraulic conductivity of the porous medium-fluid system (LT^{-1})

R, excess moisture infiltrated per day (LT^{-1})

f, drainable porosity of the porous medium (-)

The position of the water table can be written as a function of the remaining variables in the form

$$Z = f_1(t, S, h, d, K, R, f). \tag{24}$$

There are 8 variables involving two basic dimensions which should be involved in 6 dimensionless and independent pi terms. One possible set was as follows, with the term involving the water table behavior written as function of the remaining terms:

$$Z/h = f_2(S/h,\ d/h,\ Kt/h,\ R/K,\ f). \tag{25}$$

In order to have a true model of the system, each of the five pi terms on the right side of Equation 25 must be equal in model and proptotype. The first two of these terms require that geometrical similarity exist between model and prototype. The third term in effect sets up a time scale. Ligon (37) found that the last pi term, f, could be combined with the third pi term on the right of Equation 25 to form

$$\frac{K_m\ t_m}{f_m\ h_m} = \frac{Kt}{fh}$$

or,

$$\frac{t}{t_m} = n\frac{K_m}{K}\frac{f}{f_m} \qquad 26$$

where

m stands for model,

n = length scale, h/h_m, and

t/t_m = time scale.

The fourth pi term on the right of Equation 25 was used to establish a rate scale which defined the rate of fluid to be added to the model from the following analysis:

$$\frac{R_m}{K_m} = \frac{R}{K}$$

from which

$$\frac{R}{R_m} = \frac{K}{K_m},$$

or

$$R_m = Rn\frac{t_m}{t}\frac{f}{f_m}. \qquad 27$$

Equation 27 agrees with the rate scale developed by Kraijenhoff (34).

<u>Construction</u>

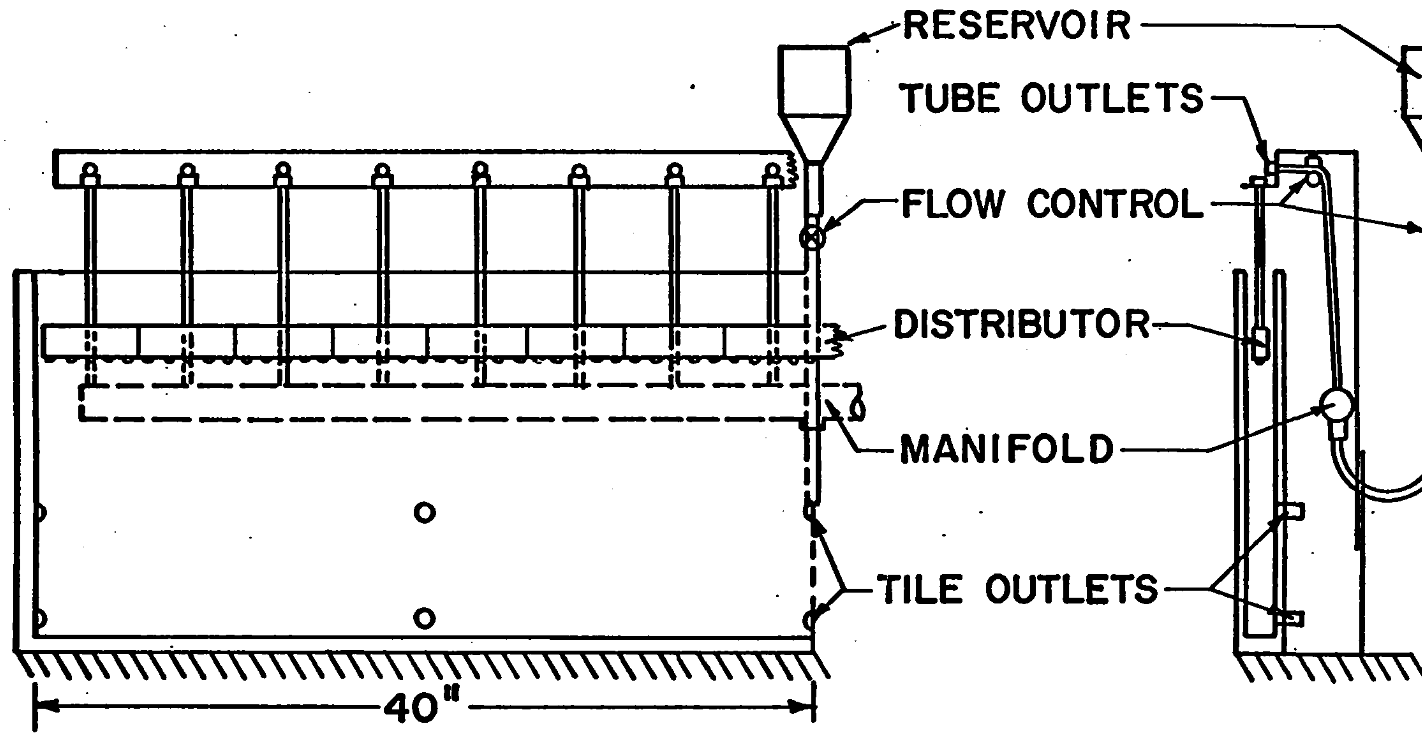
The model was a modification of the unit used by Ligon (37), and is shown schematically in Figure 4. Inside measurements of the model chamber included a width of 80 inches, a depth of 10 inches, and a thickness of 0.75 inch. The front and back were constructed of single sheets of 0.25

inch transparent plexiglass. Two rows of outlets for tile insertions were established on 20-inch centers from either end, and at distances of 0.25 inch and 2.25 inches from the bottom. Rigid plastic tubing 0.25 inch in diameter were cemented flush with the inside of the back panel such that cylindrical brass screens with 20 meshes per inch could be inserted to simulate drain tiles.

It was necessary to improvise a method for applying simulated precipitation to the surface of the model. Asseed (2) used a tank to secure a constant head, capillary tubes being connected in a horizontal position from the bottom of the tank to the top of the model. This provided a constant flow rate for arbitrarily selected time interval. Since the time scale in the present problem required a system with small time intervals, it was necessary to build an applicator which would give accurate discharge, with a uniform distribution, and facilities for immediate starting and stopping of flow. Also, since the fluid was glycerol, it was advantageous to have a system which required a minimum of fluid.

The applicator, shown in Figure 5, consisted of a reservoir for holding a measured amount of fluid until time for release. At the moment of release, a clamp was loosened from the flexible tube leading from the bottom of the reservoir. The fluid entered the center of a plexiglass cylindrical manifold 75 inches long and 1.5 inches in diameter.

Figure 4. Schematic diagram of tile drainage model with fluid applicator



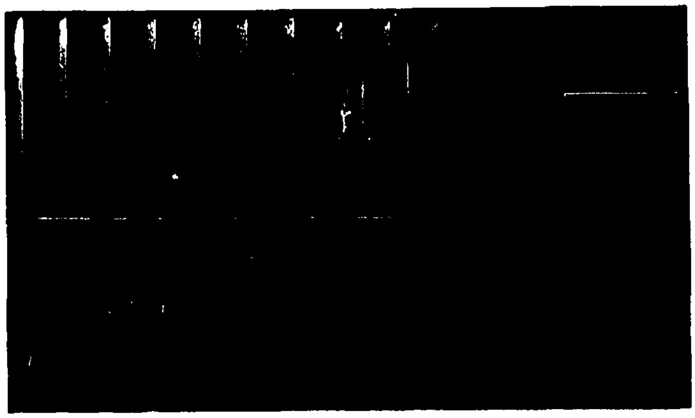
FRONT VIEW
HALF-SECTION

END VIEW OF
MODEL WITH APPLICATOR

The head from the reservoir caused fluid to exit from the manifold through sixteen 0.25-inch inside-diameter flexible tubes placed 5 inches on center along the manifold. Tube outlets were located directly above the surface of the porous medium in the model chamber. A metering device consisting of a clamp activated by two screws near the outlet of the tubes provided a means for calibrating the discharge. The flow from each tube could be very well controlled giving a uniform discharge into a distributor. Flow to the distributor from the tube outlets was conveyed through 0.25 inch vertical pipes with funnel-shaped collectors at the top of each pipe. The distributor was fabricated from two pieces of 0.25-inch thick plexiglass, two inches deep and 80 inches long. The two pieces were separated by strips two inches long and 0.25 inch square, and placed in a vertical position five inches on center from either end. The strips formed partitions such that the inflow from each vertical pipe remained separated from the flow of the pipes on either side. A bottom made of plexiglass was cemented to the distributor. Holes 0.125 inch in diameter and one inch on center were drilled in the plexiglass member on the bottom of the distributor.

Orifices made by drilling a hole 0.063 inch in diameter through the head of 0.25 inch rivets were cemented on the bottom of the distributor such that the hole through the

Figure 5. Photographs of the drainage model showing the model chamber in top photograph, fluid applicator in middle photograph, and the gauge for measuring the position of the fluid surface in the bottom photograph



rivet head was symmetrically located over the hole in the plexiglass. No further means were used to bring the orifices into calibration.

To measure the variation in distribution, five runs were made after the 16 tubes had been adjusted to the same rate of flow. A 150-ml quantity of fluid was released from the reservoir, and the discharge from individual tubes was collected. An analysis of variance indicated no significant difference at the 0.05 probability level. The data from the calibration test are given in Table 13 in Appendix C.

A similar evaluation was made for the discharge from the five orifices beneath each of the compartments which made up the distributor. One cell at the time was run by pouring into the center of the cell a 50-ml quantity of fluid and measuring the discharge from each orifice. The average values of the five runs for each orifice were grouped over 13 compartments. These values are given in Table 14 in Appendix C. The standard deviation of the means was 0.05 ml. There was a significant difference at the 0.05 probability level. The calculated least significant difference at the 0.05 probability level was 0.142 ml. The average values for the 5 orifices in order of their relative positions were 1.821, 1.905, 1.999, 1.894, and 1.836 ml. The difference between the highest and the lowest was 9.41 percent of the average value. It can be observed that the highest value was at the center of the range with the lowest values at each end. This was due to the slightly higher head at the center of the cell where the fluid entered.

Evaluation of Water Table Behavior by Model

It was desired to see if Ligon's findings (37) concerning the modeling of hydraulic conductivity applied as well for tile drains as was found for open-ditch drains. This investigation was carried out using the moisture excess for the year 1960. Glycerol and glass beads 2 mm in diameter were used first. The hydraulic conductivity and drainable porosity were measured in the same manner described by Ligon, and found to be 0.78 inch per minute and 35.8 percent, respectively, at a temperature of 72°F. In order to evaluate the time scale, arbitrary field conditions were selected consisting of a hydraulic conductivity of 3.7 inches per hour and a drainable porosity of 8 percent. A length scale, n, of 24 was also selected. Applying these values to

$$t/t_m = n\ (f/f_m)\ (K_m/K)$$

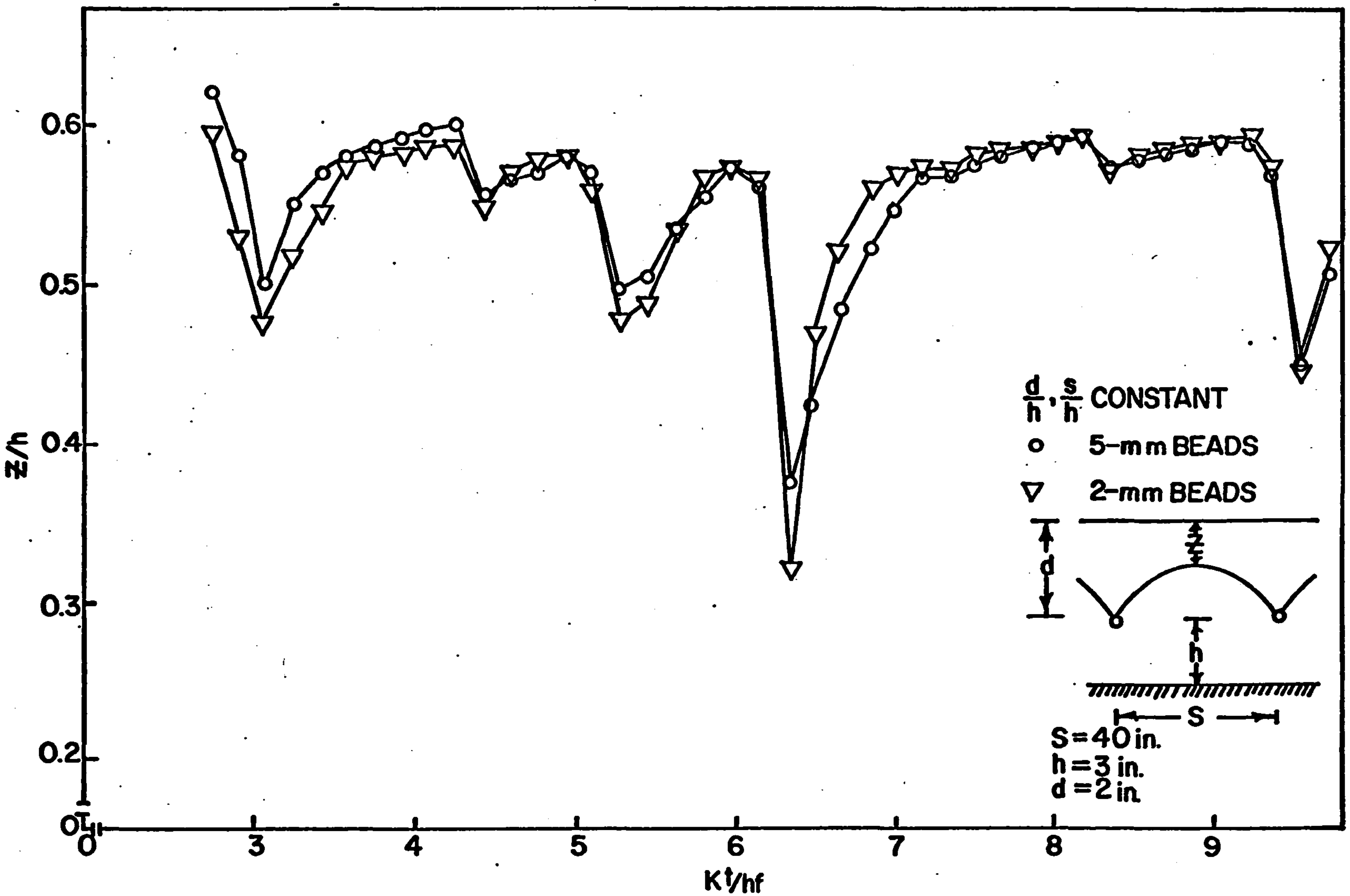
the relationship was such that when t was one day, t_m was 23.5 minutes.

The second step was to "model" the model. This was accomplished by using 5-mm beads in place of the 2-mm beads. Ligon accomplished the same thing by changing the viscosity of the fluid. The new hydraulic conductivity was 3.18 inches per minute with a drainable porosity of 37.4 percent. The length scale between the 2 models was unity. This gave a time ratio of 3.8 to 1 when comparing the 2-mm beads to the 5-mm beads, which gave a time of 6 minutes in the 5-mm beads for the comparable 23.5 minutes in the 2-mm beads.

The excess determined by the water balance for the months April, May, and June of 1960 were used in the preliminary investigations. The excess was applied to the model for each of the above time scales. The resulting curves are shown in Figures 6, 7, and 8 for model-tile spacings of 40, 80, and 160 inches. The ratio of S/h was 13.3, 26.6, and 53.3 respectively, for the spacings. The depth of tile below the surface of the beads was two inches, and the depth to the impervious layer below the tile centers was three inches. The dimensionless plots of Z/h vs (Kt)/(hf) were made for S/h constant between the models for a given spacing, and d/h constant between models and all spacings. The term R/K was held relatively constant in that the fluid was applied to the surface through a reservoir which had a variable head of two inches in a total head of 11 inches. A constant-temperature chamber was used and the temperature was controlled at 72°F.

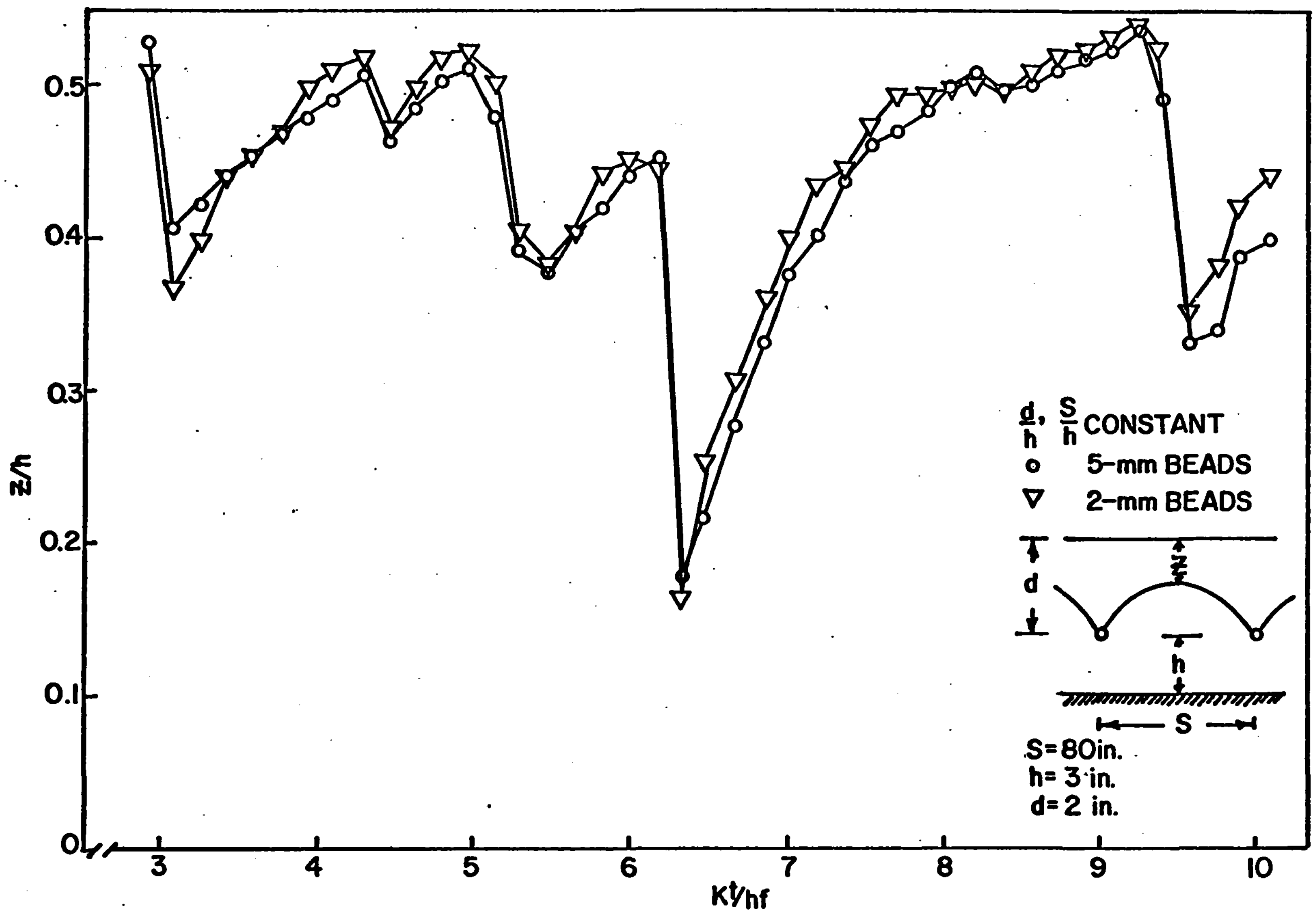
There was a tendency for the curves established from the 2-mm beads to fall slightly lower than the curves obtained from the 5-mm beads during times of peak infiltration. The curves agreed very well otherwise, and it was concluded that

Figure 6. Dimensionless water table fluctuation for S/h = 13.3 with the hydraulic conductivity, K, and the drainable porosity, f, varied between runs (See Table 4 for excess schedule and time - Kt/hf relationship)



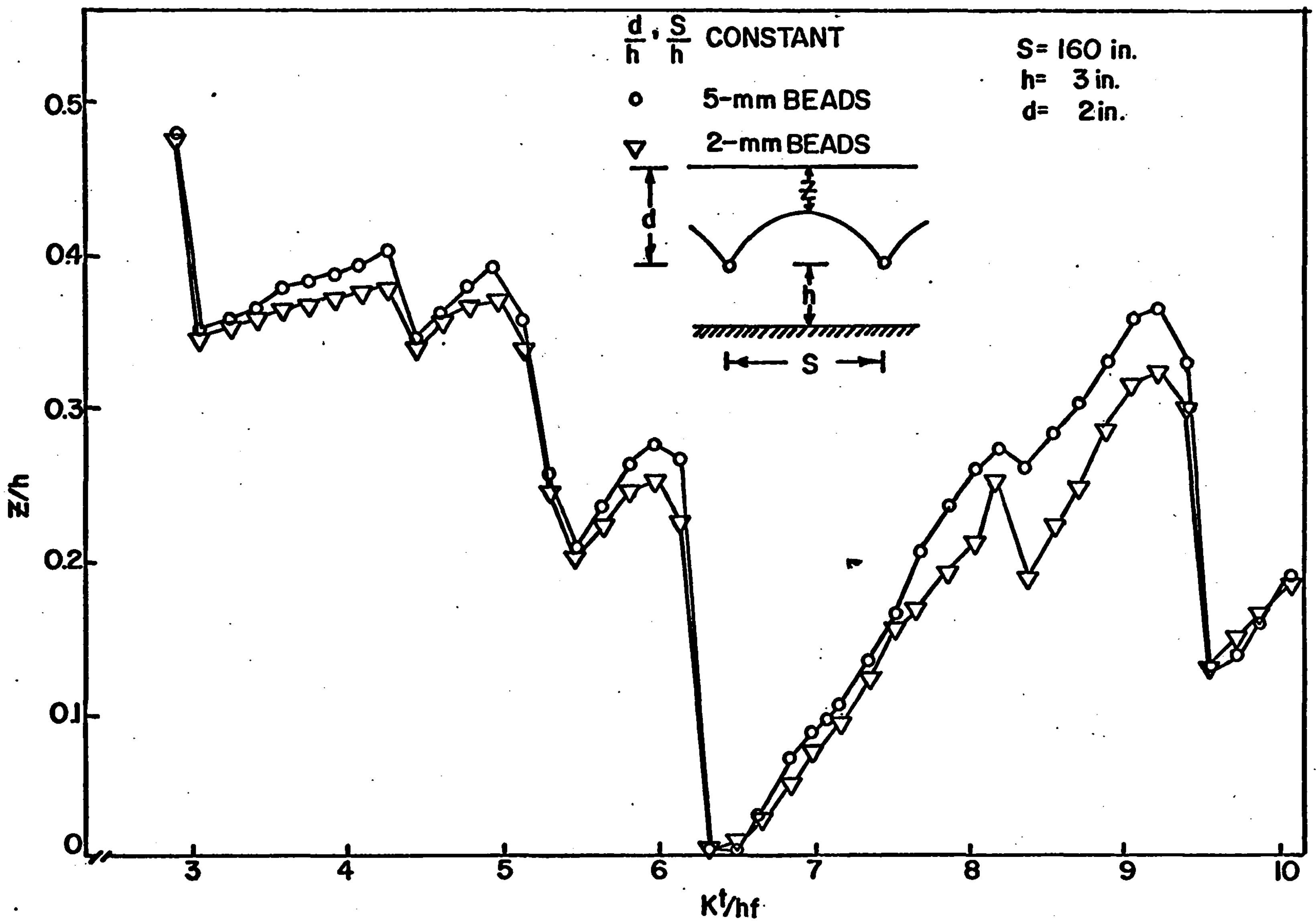
$\frac{d}{h}, \frac{s}{h}$ CONSTANT
o 5-mm BEADS
▽ 2-mm BEADS
S = 40 in.
h = 3 in.
d = 2 in.
d
h
S
z
z/h
K t/hf
0.6
0.5
0.4
0.3
0.2
0
0
3
4
5
6
7
8
9

Figure 7. Dimensionless water table fluctuation for $S/h = 26.6$ with the hydraulic conductivity, K, and the drainable porosity, f, varied between runs (See Table 4 for excess schedule and time - Kt/hf relationship)



0.5
0.4
0.3
0.2
0.1
0
z/h
3
4
5
6
7
8
9
10
Kt/hf
d/h, S/h CONSTANT
o 5-mm BEADS
▽ 2-mm BEADS
z
d
h
S
S=80in.
h= 3 in.
d= 2 in.

Figure 8. Dimensionless water table fluctuation for S/h = 53.3 with the hydraulic conductivity, K, and the drainable porosity, f, varied between runs (See Table 4 for excess schedule and time - Kt/hf relationship)



$\frac{d}{h}$, $\frac{S}{h}$ CONSTANT
5-mm BEADS
2-mm BEADS
S= 160 in.
h= 3 in.
d= 2 in.
d
h
S
z
z/h
K^t/hf
0.5
0.4
0.3
0.2
0.1
0
3
4
5
6
7
8
9
10

variation of the hydraulic conductivity, K, which resulted from a change in the size of the porous medium, had no effect on the relationship between the dimensionless terms Z/h and (Kt)/(fh). However, an attempt to obtain a single functional relationship of the variables was not successful.

Table 4. Excess schedule and time - Kt/hf relationship as used in Figures 6, 7, and 8

Date, 1960	Excess, in.	Kt/hf
April 16	0.61	2.73
17	0.72	2.90
25	0.34	4.26
29	0.23	4.95
30	0.63	5.12
May 1	0.30	5.29
5	0.13	5.97
6	1.80	6.14
18	0.17	8.19
24	0.31	9.21
25	1.10	9.38

Analytical Methods

It was not readily apparent to this writer how the above procedure could be adapted to use in drainage design without excessive time being spent to secure data which would be too restricted for wide practical use. It was at this point that a search was made for analytical methods which could be adapted to computer analysis such that a wide range of conditions could be investigated.

Kraijenhoff's equation

As pointed out in the review of literature, Kraijenhoff (35) developed a method for computing the elevation of the water table at the midpoint between tile lines. When 4 terms (n = 1, -3, 5, -7) were used in expanding Equation 1, the general Equation as given by Equation 8 changed to:

$$\begin{aligned} Y_t = {} & Y^{*}_{t-1}[\exp(-1/j)] + [4P_t/(f\Pi)]\ (j)\ [1-\exp(-1/j)] \\ & - Y^{**}_{t-1}[\exp(-9/j)] - [4P_t/(f\Pi)]\ (j/27)\ [1-\exp(-9/j)] \\ & + Y^{***}_{t-1}[\exp(-25/j)] + [4P_t/(f\Pi)]\ (j/125)\ [1-\exp(-25/j)] \\ & - Y^{****}_{t-1}[\exp(-49/j)] - \\ & - [4P_t/(f\Pi)](j/343)\ [1-\exp(-49/j)] + r_{4_t}. \end{aligned} \qquad 28$$

The variables were defined previously. The term, r_{4_t}, was defined as

$$r_{4_t} = [4P/(f\Pi)]\ (j) \sum_{n=1,-3,5}^{\infty} 1/n^3 - [1-(1/27) + (1/125) -(1/343)], \qquad 29$$

and by letting

$$\sum_{n=1,-3,5}^{\infty} (1/n^3) = \Pi^2/32,$$

then

$$r_{4_t} = [4P/(f\Pi)]\ (j)\ (\Pi^3/32)[-1+(1/27)-(1/125) + 1/343)].$$

The above equations were used to calculate the position of the water table midway between the outflow drains during the months of April, May, and June for the year 1960. It was assumed that a given time distribution of percolation into the phreatic zone could be approximated by a succession of intervals which had constant percolation rates.

Before calculations could be achieved, it was necessary to evaluate the reservoir coefficient, j. This parameter was a function of the drainable porosity, drain spacing, hydraulic conductivity, and the average depth of the impermeable layer below the water table. This involved the assumption that the height of the water table above the tile was small with respect to the total depth of groundwater flow, which means that the value was considered constant. This simplification neglected the effect of convergence of flow towards the drain. The error would be more pronounced as the depth of flow throughout the aquifer increased (7). It was further observed that as the depth of flow decreased, the assumption of a time-constant thickness of the acquifer was not met. It was assumed that these problems could be solved by using the recession curve developed from the model for the respective variables used in the analytical procedure. The initial point of the recession curve (see review of literature) was determined, and the angle between the curve and the abscissa was defined. These curves are presented in Figure 9, and the values of the reservoir coefficient for the various drain spacings are given in Table 5.

Table 5. Reservoir coefficients developed from model for various tile spacings where the depth to the impervious layer was 6 feet, the hydraulic conductivity was 1.5 inches per hour, and the drainable porosity was 12 percent.

Tile spacing ft.	Reservoir coefficient, days
40	10.4
80	15.6
160	34.7
320	82.0

A graph comparing the results of the water table behavior obtained from the model to the results obtained from the Kraijenhoff equation for conditions as given in Table 5 for the 80-foot tile spacing are presented in Figure 10. Moisture excess for the time interval April 1 to June 10, 1960 were used to develop this graph. It can be observed that the calculated values were consistently higher than the values observed in the model, especially on the day which excess occurred. As tile spacings were increased to 160 feet and 320 feet, the difference between the two methods of prediction remained about the same on days of no excess, but the discrepancy was unreasonable on the days of excess. For instance, when using a tile spacing of 320 feet, the water table observed in the model moved to the ground surface on May 25, but the value calculated by Kraijenhoff's equation on that day would have raised the water table 7.1 feet above the ground surface if

Figure 9. Drawdown from complete saturation in model using 5-mm glass beads and glycerol

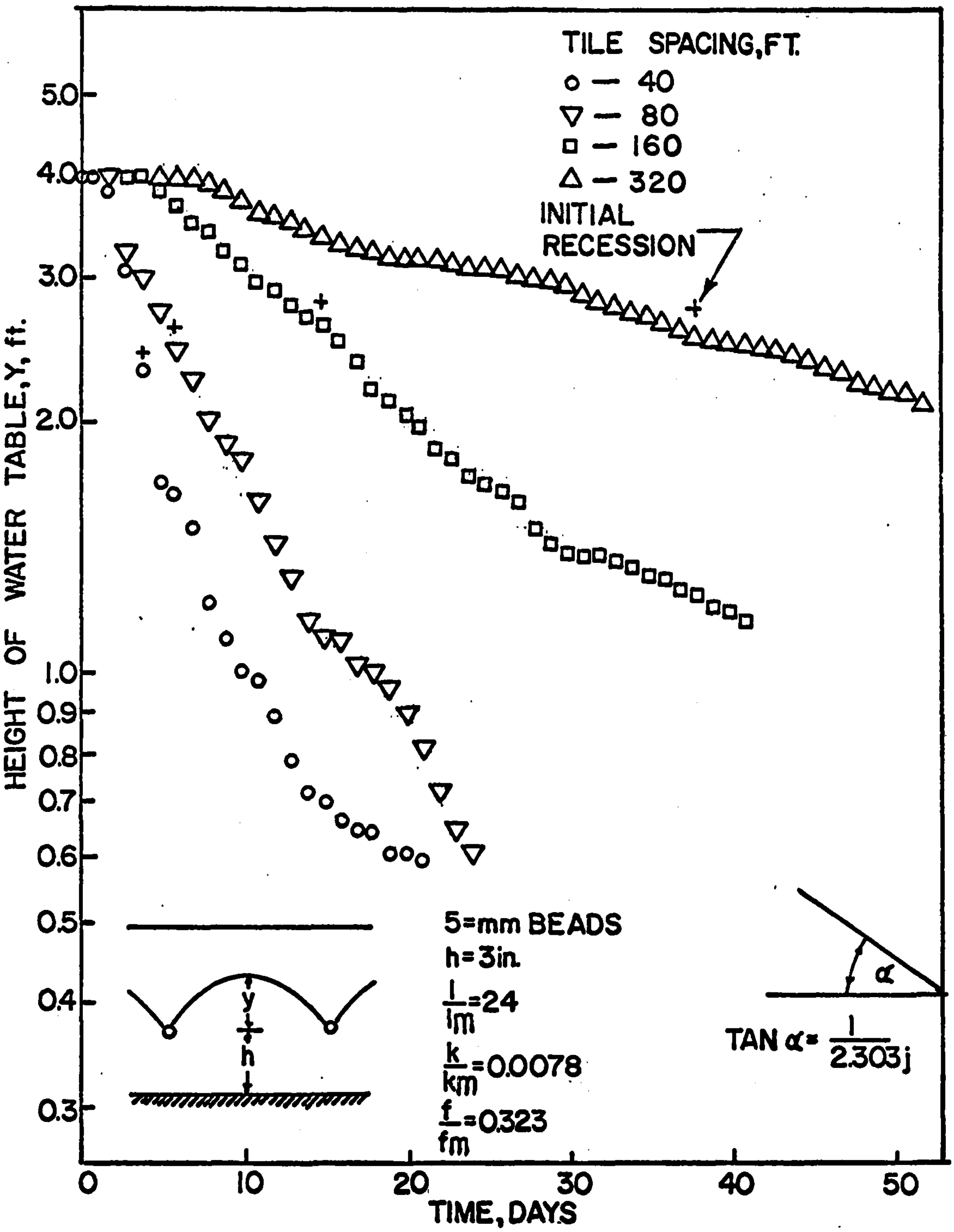
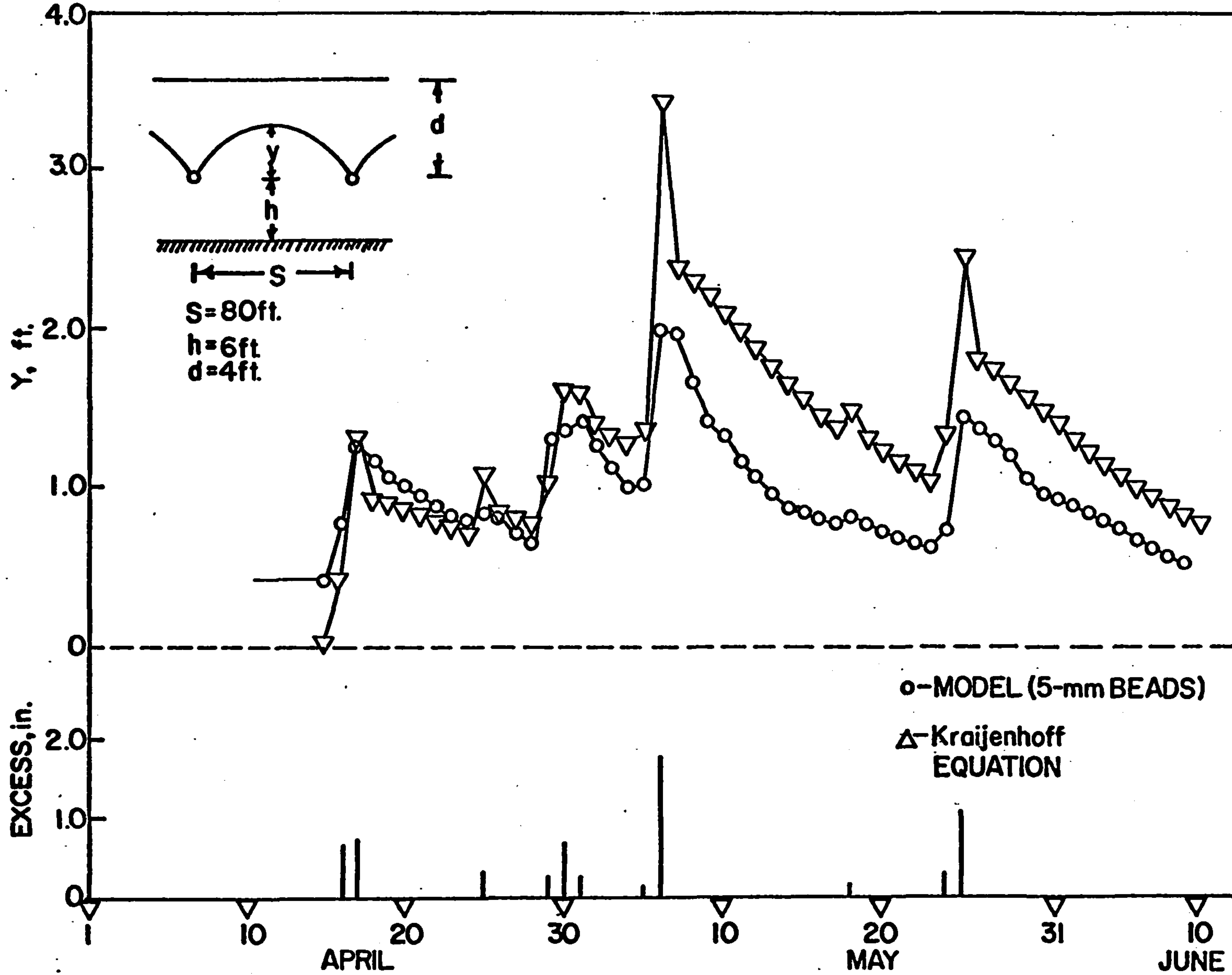

TILE SPACING, FT.
o — 40
▽ — 80
□ — 160
△ — 320
INITIAL RECESSION
HEIGHT OF WATER TABLE, Y, ft.
5.0
4.0
3.0
2.0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
5=mm BEADS
h=3in.
l/lm=24
k/km=0.0078
f/fm=0.323
y
h
TAN α = 1/2.303j
α
0
10
20
30
40
50
TIME, DAYS

Figure 10. Excess applied to model and to Kraijenhoff's equation for the time interval April 1 to June 10, 1960, a tile spacing of 80 feet, a tile depth of 4 feet, and a depth of 6 feet to the impervious layer below the plane of the tile drains


4.0
3.0
2.0
1.0
0
Y, ft.
d
y
h
S
S=80ft.
h=6ft.
d=4ft.
EXCESS, in.
2.0
1.0
0
o-MODEL (5-mm BEADS)
Δ-Kraijenhoff EQUATION
1
10
20
30
10
20
31
10
APRIL
MAY
JUNE

there had been a sufficient depth of soil to accomodate this condition. In reality, with the soil depth restricted to four feet above the tile, the calculated value would be interpreted as yielding a depth of ponded water equivalent to the 7.1 feet of saturated soil.

The discrepancy between the model and Kraijenhoff's equation was believed to be the result of the component method of prorating the effect of the excess from the previous day. It appears that this procedure did not satisfy the principle of superposition for breaking down a complicated time distribution of percolation into increments of steady percolation on the day which excess occurred, and the two following days. In comparing the model results to the analytical results, no correction was made for the capillary fringe in the model.

Van Schilfgaarde's equation

The height of the water table measured from the plane of the drains at a point midway between two drains was investigated by the use of Equation 21:

$$
\begin{aligned}
Y_N = A/f \Big\{ & P_1[e^{-(N-1)A} - e^{-N/A}] \\
& + P_2\,[e^{-(N-2)/A} - e^{-(N-1)/A}] \\
& \cdot \\
& \cdot \\
& \cdot \\
& + P_N[e^{-(N-N/A)} - e^{-(N-N+1)/A}] \Big\}
\end{aligned}
\qquad 21
$$

where A was defined as the ratio fFCS/K.

The factor A was considered equivalent to Kraijenhoff's reservoir coefficient. It combined the geometric restraints of the system, as fixed by F and S, with the soil properties, f and K, into one constant.

The meaning of P_N in Equation 21 was modified to reflect the net accretion rate rather than the precipitation rate. This followed the procedure used in Kraijenhoff's equation in that the excess as determined by the water balance was used as input on the respective days.

Values for F were calculated using Equation 11. The computer program for these calculations, and the values of F for four tile spacings, one tile diameter, and several depths to the impervious layer are given in Appendix D.

The value of C in the factor A was taken as 0.80. Bouwer (6) has shown that C is about 0.8 for Y/S values ranging from 0.02 to 0.08 when the depth to the impervious layer is relatively small. This would be the case for drain spacings of 40 feet or greater, and a water table less than one foot above the tile drains, measured at the midsection. When the value of Y/S exceeded 0.15, the value of C was about one. This would be approached when a water table of three feet or more prevailed with 20-foot tile spacings.

Values for F and C, as determined above, were used in Equation 21 to calculate the water table behavior for a K-value of 1.5 inches per hour, an f-value of 12 percent, a tile depth of 4 feet, a depth to the impervious layer below the tile of 6 feet, and tile spacings of 40, 80, 160, and 320 feet. Excess for the time period April 1 to June 10 were used from the year 1960. The calculated water tables were compared to the observed water tables in Figures 11 through 14. The observed values were taken from the glass bead-glycerol model, and these values have been corrected by reducing the actual observed water table in the model by a factor of 0.2 inch. This consideration became apparent when it was observed that the calculated water table was consistently below the modeled water table by a factor of about 0.5 foot. This difference was about equal to the capillary fringe in the model. Ligon (37) reported a fringe effect of 0.3 inch in the model when 2-mm beads were treated with a silicone material. Grover (21) found that when the beads were treated such that a 90-degree wetting angle prevailed, there still was a resistance to motion of the surface of saturation which gave a "pseudo" capillary fringe of approximately 0.75 cm of glycerol when the fluid was draining from the glass beads. Ligon _et al_. (40) compared theoretical curves of dimensionless drawdown with experimental data. It was found that when the capillary fringe was considered in the theoretical development, there was better agreement between theory and experimentation than was the case when the capillary fringe was disregarded in theory.

Figure 11. Water table behavior for 40-foot tile spacing as found by model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 6-foot depth from the tile to the impervious layer

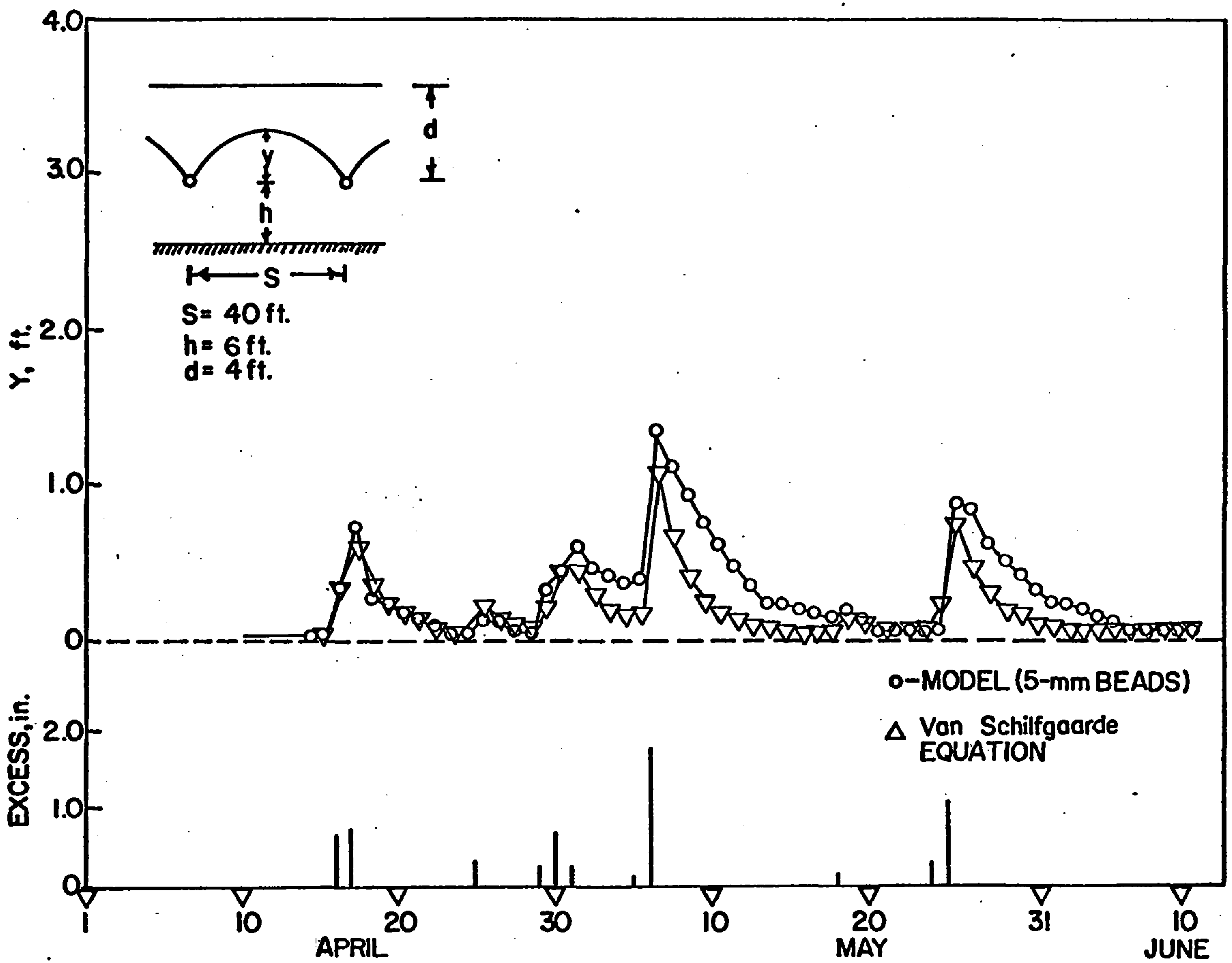
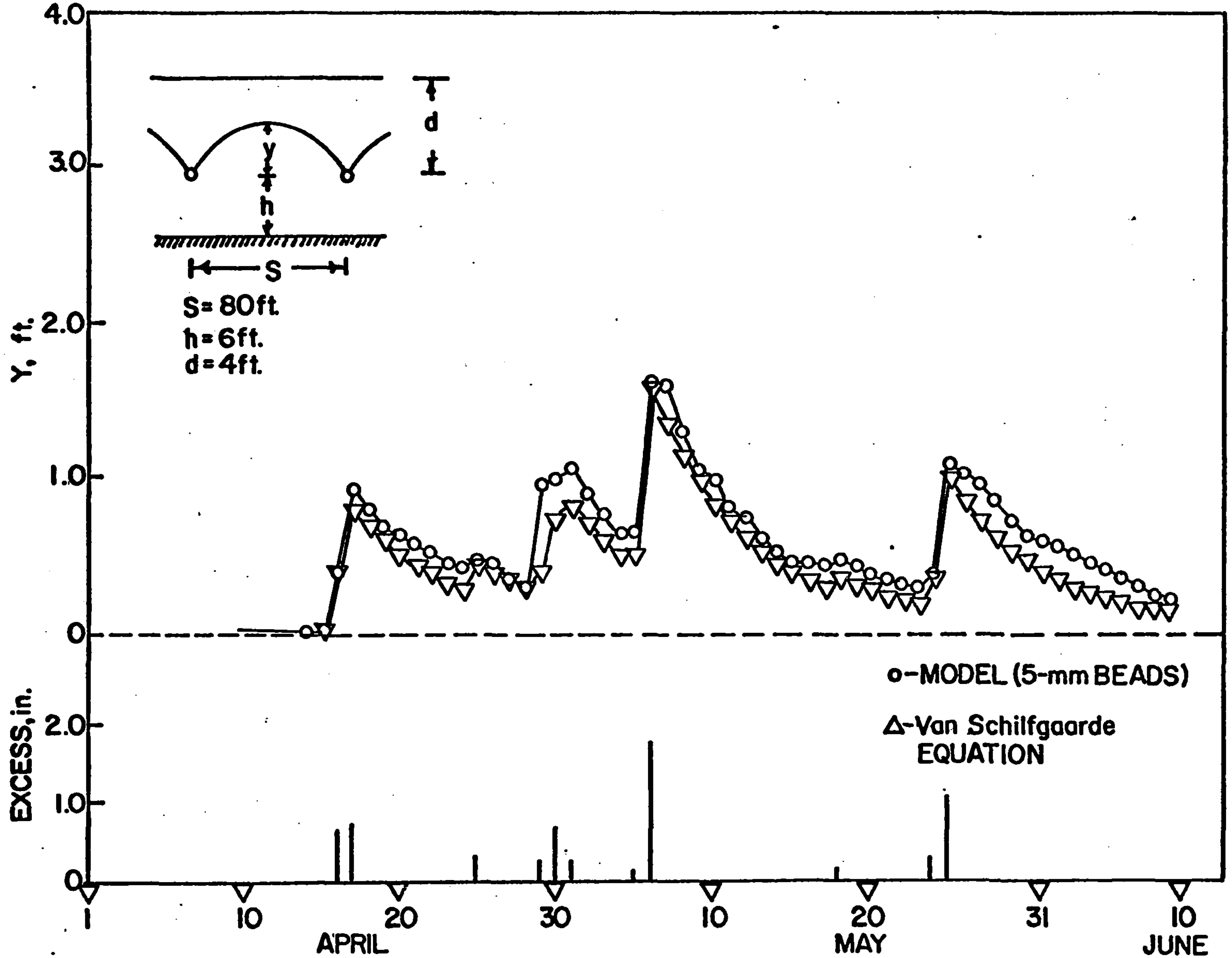
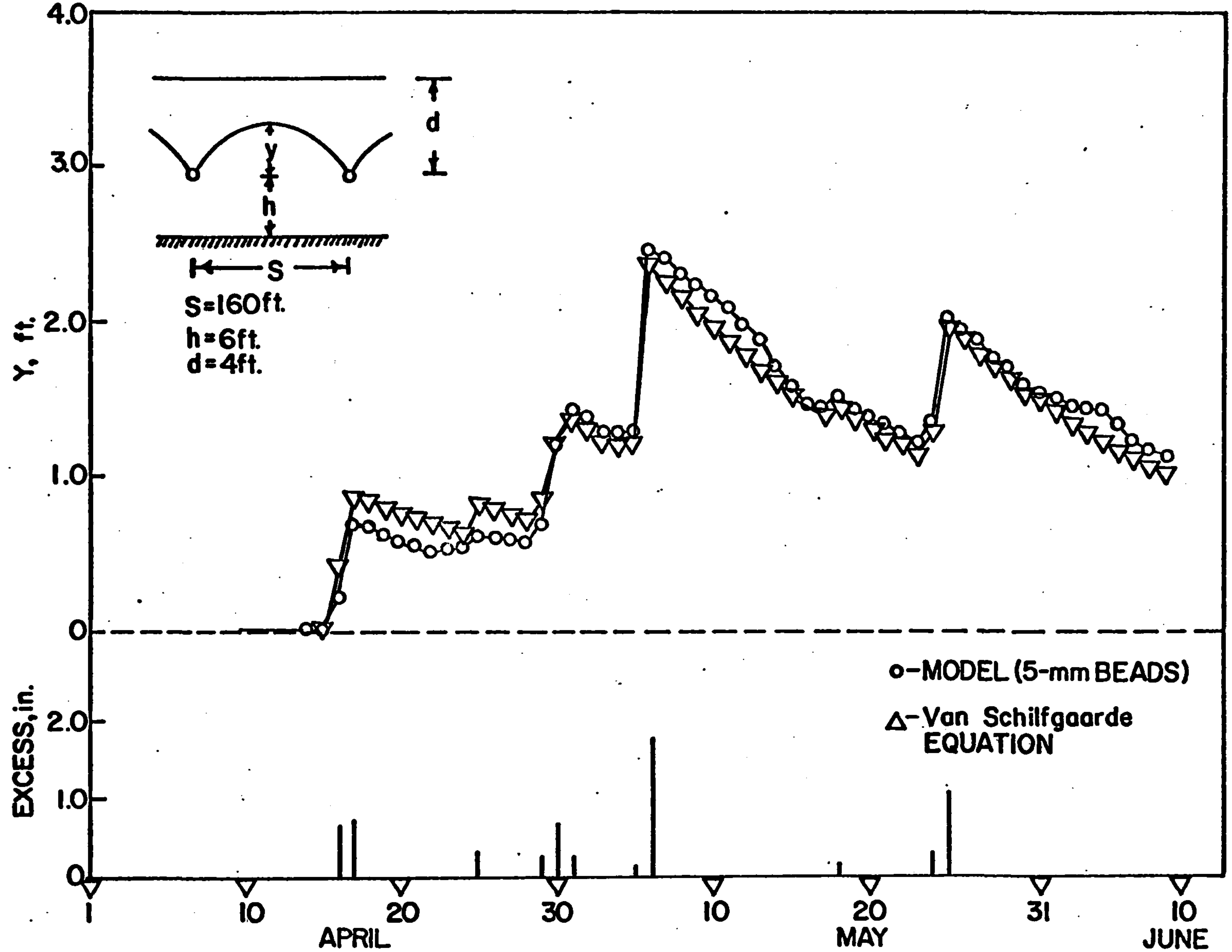

4.0
3.0
2.0
1.0
0
Y, ft.
d
y
h
S
S= 40 ft.
h= 6 ft.
d= 4 ft.
o-MODEL (5-mm BEADS)
△ Van Schilfgaarde EQUATION
EXCESS, in.
2.0
1.0
0
1
10
20
30
10
20
31
10
APRIL
MAY
JUNE

Figure 12. Water table behavior for 80-foot tile spacing as found by model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 6-foot depth from the tile to the impervious layer



S = 80 ft.
h = 6 ft.
d = 4 ft.
Y, ft.
EXCESS, in.
o–MODEL (5-mm BEADS)
Δ–Van Schilfgaarde EQUATION
APRIL
MAY
JUNE

Figure 13. Water table behavior for 160-foot tile spacing as found by model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 6-foot depth from the tile to the impervious layer



4.0
3.0
2.0
1.0
0
Y, ft.
d
y
h
S
S=160ft.
h=6ft.
d=4ft.
EXCESS, in.
2.0
1.0
0
o-MODEL (5-mm BEADS)
Δ-Van Schilfgaarde EQUATION
1
10
20
30
10
20
31
10
APRIL
MAY
JUNE

Figure 14. Water table behavior for 320-foot tile spacing as found by model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 6-foot depth from the tile to the impervious layer

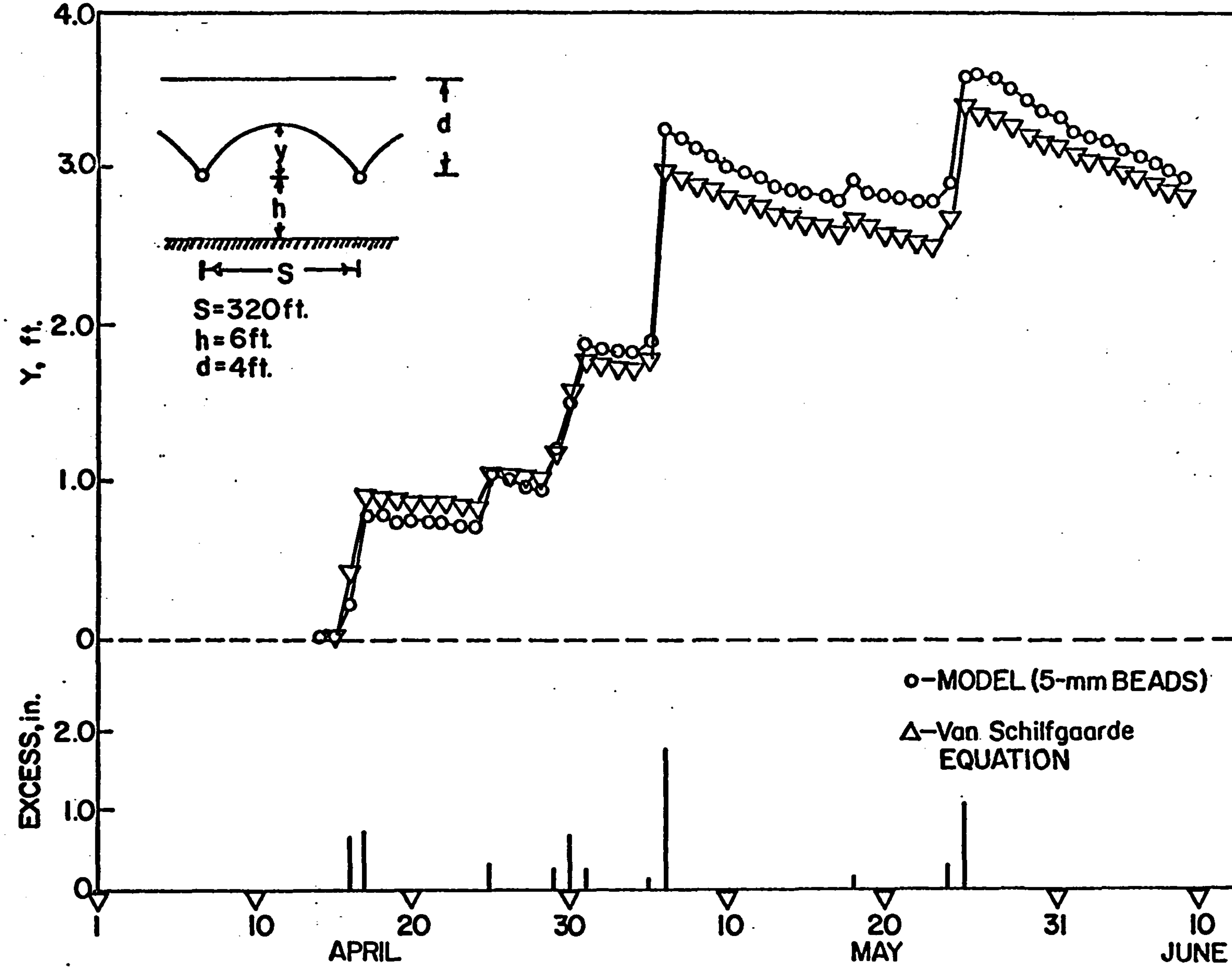

4.0
3.0
2.0
1.0
0
Y, ft.
d
y
h
S
S=320 ft.
h=6 ft.
d=4 ft.
o–MODEL (5-mm BEADS)
Δ–Van Schilfgaarde EQUATION
EXCESS, in.
2.0
1.0
0
1
10
20
30
10
20
31
10
APRIL
MAY
JUNE

Figure 15. Water table behavior for 40-foot tile spacing as found by model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 0.50-foot depth from the tile to the impervious layer

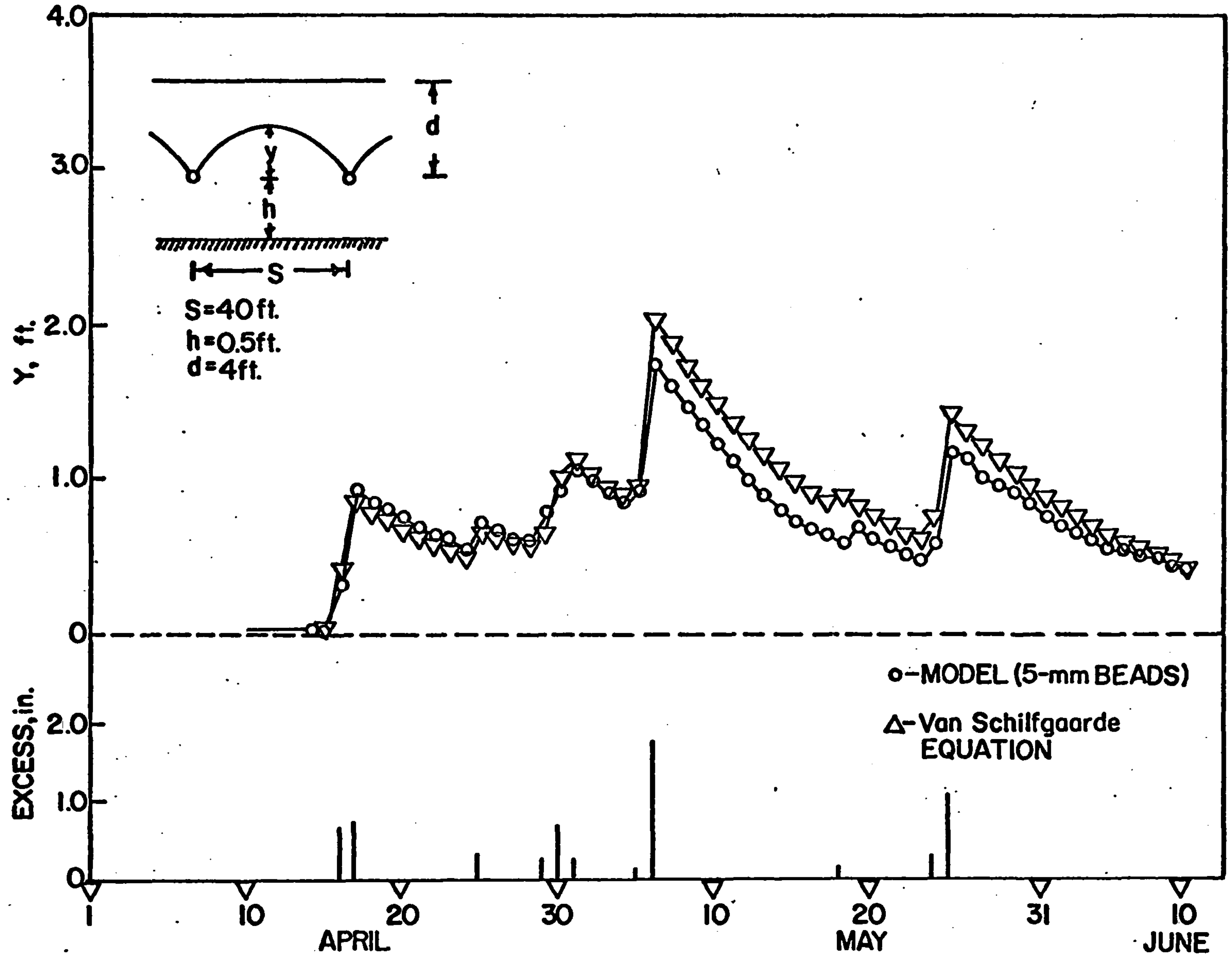
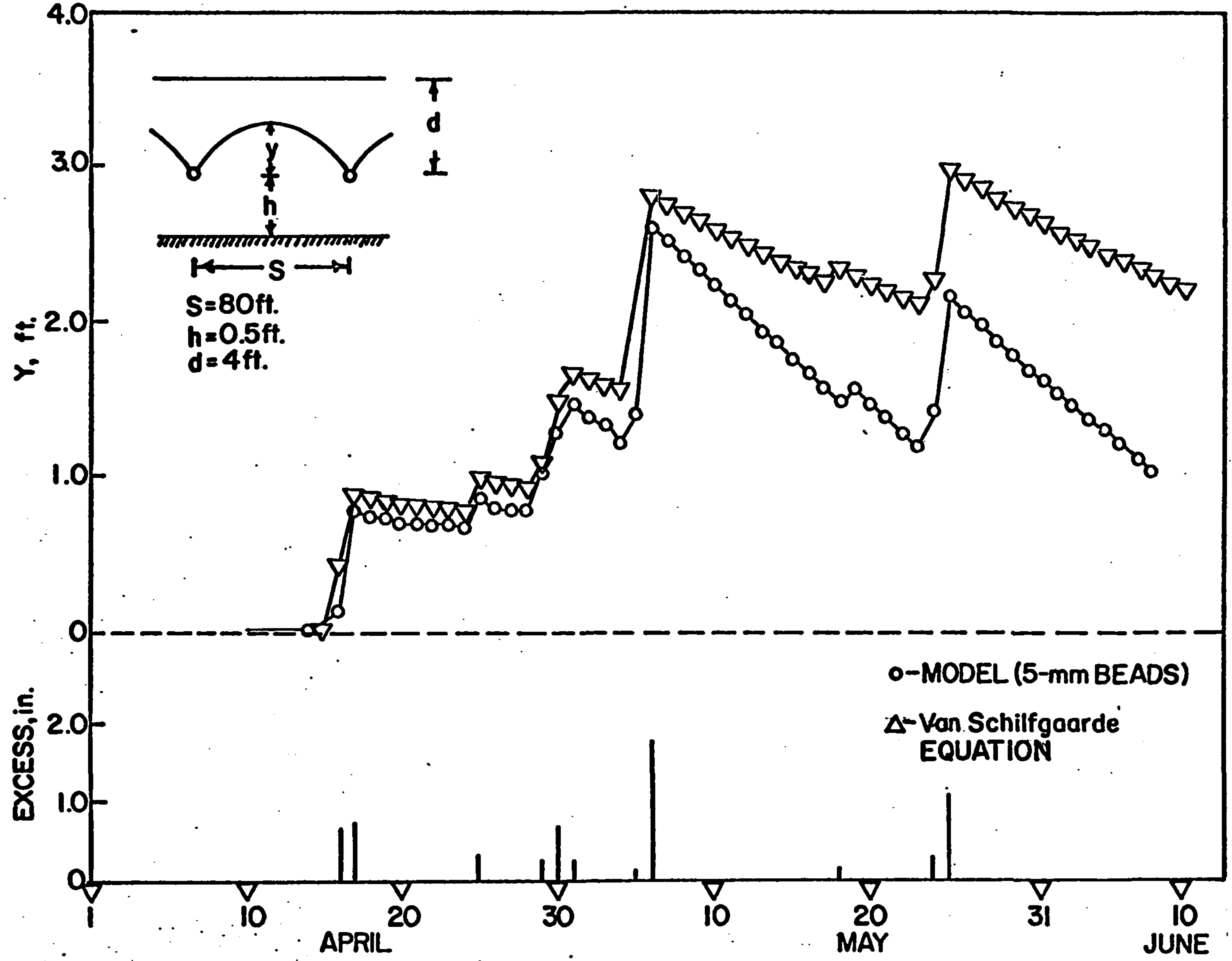
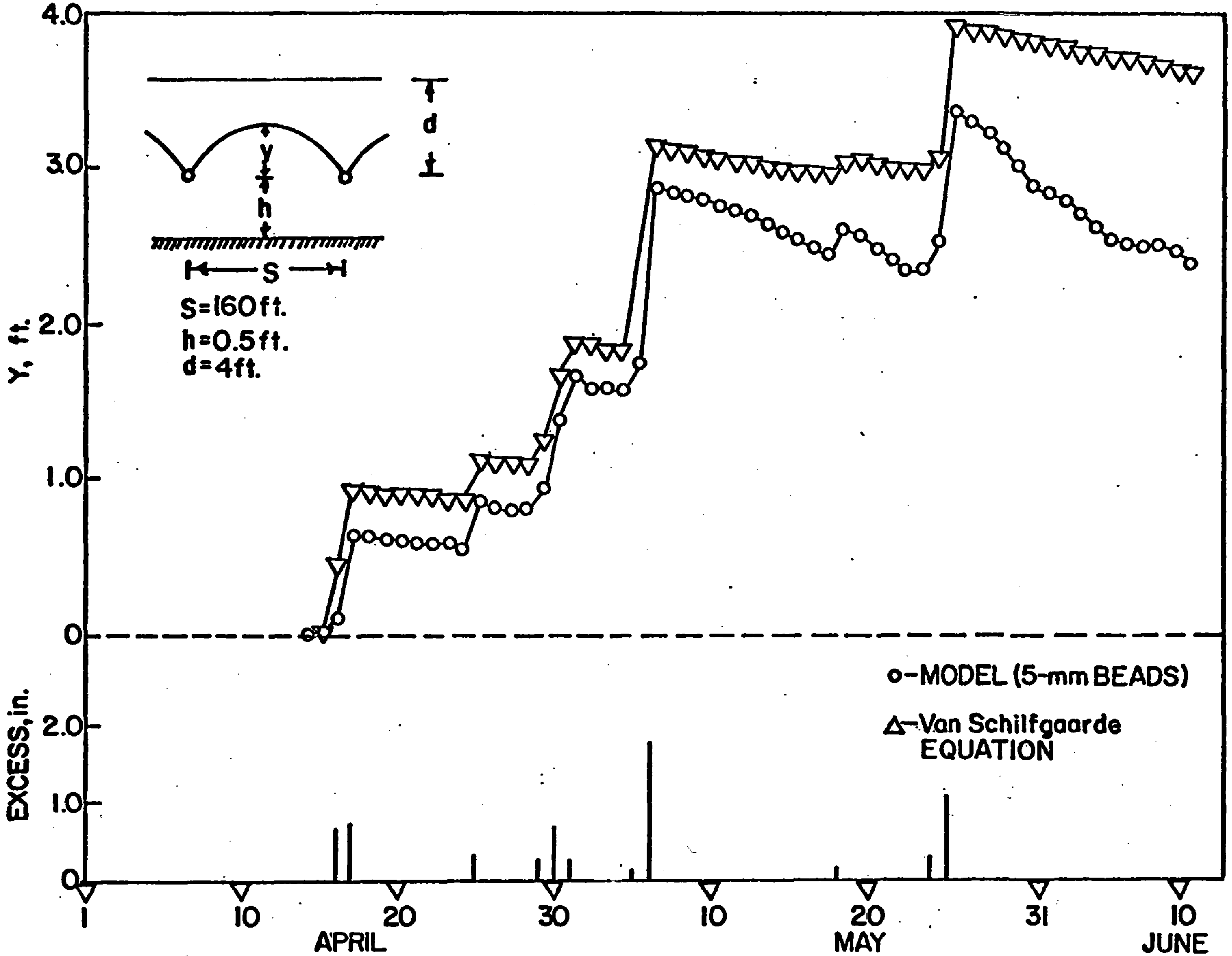

4.0
3.0
2.0
1.0
0
Y, ft.
d
y
h
S
S=40 ft.
h=0.5 ft.
d=4 ft.
o-MODEL (5-mm BEADS)
Δ-Van Schilfgaarde EQUATION
EXCESS, in.
2.0
1.0
0
1
10
20
30
10
20
31
10
APRIL
MAY
JUNE

Figure 16. Water table behavior for 80-foot tile spacing as found by glass bead-glycerol model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 0.50 foot depth from the tile to the impervious layer



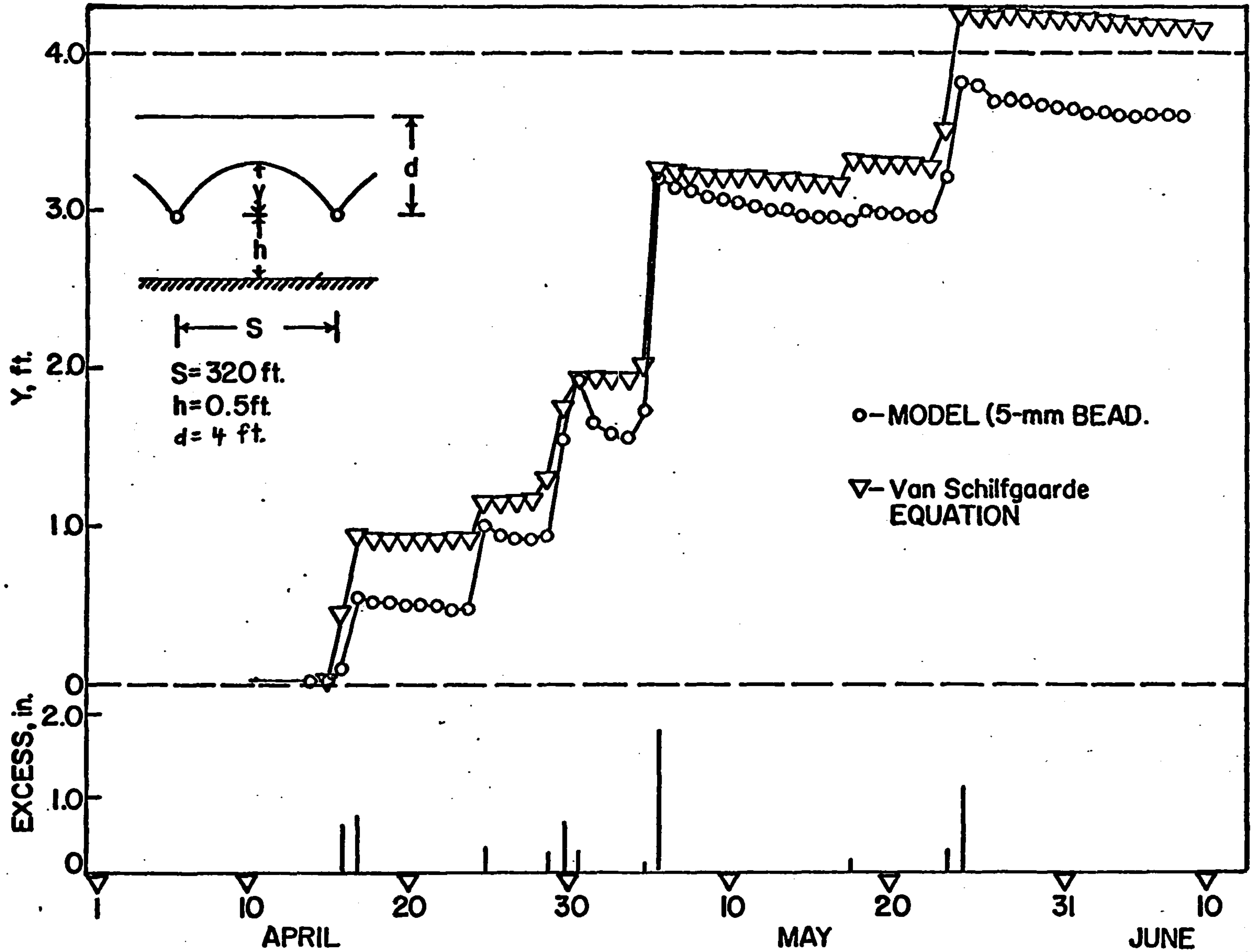
4.0
3.0
2.0
1.0
0
Y, ft.
d
y
h
S
S=80ft.
h=0.5ft.
d=4ft.
EXCESS, in.
2.0
1.0
0
o-MODEL (5-mm BEADS)
Δ-Van Schilfgaarde EQUATION
1
10
20
30
10
20
31
10
APRIL
MAY
JUNE

Figure 17. Water table behavior for 160-foot tile spacing as found by glass bead-glycerol model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 0.50 foot depth from the tile to the impervious layer



Y, ft.
EXCESS, in.
S=160 ft.
h=0.5 ft.
d=4 ft.
d
y
h
S
o-MODEL (5-mm BEADS)
△-Van Schilfgaarde EQUATION
APRIL
MAY
JUNE

Figure 18. Water table behavior for 320-foot tile spacing as found by glass bead-glycerol model (corrected for capillary fringe) and van Schilfgaarde's equation using excess from the period April 1 to June 10, 1960, and a 0.50-foot depth from the tile to the impervious layer


4.0
3.0
2.0
1.0
0
Y, ft.
d
y
h
S
S=320 ft.
h=0.5ft.
d= 4 ft.
o– MODEL (5-mm BEAD.
▽– Van Schilfgaarde EQUATION
EXCESS, in.
2.0
1.0
0
1
10
20
30
10
20
31
10
APRIL
MAY
JUNE

Since no compensation was made for the capillary fringe in the development of Equation 21, it was assumed that the capillary fringe in the model should be deleted from the observed water table values in order to have results comparable to the calculated values. The average value of the capillary fringe in the model at complete drawdown was 0.2 inch. When a length scale of 24 was used, this amounted to a value of 0.4 foot which was the amount subtracted from the observed model values. The correction for capillary fringe appears to be valid considering the consistent agreement between model and analytical results for all tile spacings.

Effect of Shallow Impervious Layer

It would have been impractical to consider all possible conditions which may exist in field conditions relative to the location of the impervious layer. Therefore, it was decided to place greater emphasis on the condition where the impervious layer existed at a depth of about twice that of the installed drain lines. However, some consideration was given to an impervious layer located at a lesser depth. It was concluded that a depth equivalent to one tile diameter below the center of the drain should be investigated. Since this problem was limited to 0.50-foot diameter tile, the shallower depth to the impervious layer was fixed at 0.50 foot below the center of the drain.

The same conditions as previously used for the deeper impervious layer were applied to the model, and to van Schilfgaarde's equation for the case of the shallower impervious layer. The curves for the resulting water table fluctuations are given in Figures 15 through 18 for tile spacings of 40, 80, 160, and 320 feet respectively. The results from the model were corrected for the effect of the capillary fringe by subtracting 0.2 inch from the observed model values. The curves for the 40-foot spacing show close agreement. The agreement was not as good for the 80-foot spacing. The curves remained together until the water table reached a height greater than 2.5 feet above the tile measured at the midsection. Then the observed water table in the model descended at a faster rate than the calculated water table. The same trend prevailed for the 160-foot spacing. However, the agreement appeared to improve for the 320-foot spacing. It should be noted that no provisions were made for an upper limit of the water table in the theoretical analysis. That is the reason for an indicated water table higher than the ground surface in Figure 18.

The discrepancy between theory and experimentation for the shallow impervious layer may be due to several factors. Talsma and Haskew (10) found that the physical response of water tables supported several theoretical analyses where the physical assumptions underlying that analysis were reasonably met. Some caution appeared to be necessary for design in cases where there was an impervious layer at a small distance below the tile. Luthin and Worstell (44) reported that the critical depth of the impervious layer below the tile drain was about two feet. Below this depth the rate of flow into a tile line had a linear relationship with the water table at the midpoint between the drains, but when the impervious layer was closer than two feet the relationship was no longer linear.

The values of F in the factor A increased very rapidly as the distance to the impervious layer became smaller than two feet below the tile. This can be observed from the calculated F values in Appendix D. A large F value for a shallow impervious layer tends to increase the predicted water table height at some relationship other than a linear relationship, as mentioned earlier. This was confirmed by Toksoz and Kirkham (63) since it was stated that for very large values of S/h the convergence loss near the drain became negligible when compared with loss of head due to horizontal flow, thereby causing Y/h to vary as the square of S/h. It was concluded that the differences between the analytical and the model results were not substantial enough to invalidate the use of Equation 21 for calculating water table heights.

Drainage Treatments

Equation 21 was programmed on an IBM 7074 computer, and the excess as previously determined were substituted for P_N as the net accretion into the soil. The depth of tile was limited to four feet. Tile spacings of 40, 80, 160 and 320 feet were used. Hydraulic conductivities of 1, 2, 4, and 8 feet per day were selected. Each of the above combinations were computed using a 4-foot depth to the impervious layer below the plane of the tile drains. This gave 16 treatment combinations.

In addition to the above treatments, a shallow impervious layer 0.5 foot below the drains was used in conjunction with the hydraulic conductivity of two feet per day for the four tile spacings. A summary of these conditions is given in Table 6. A total of 20 combinations were investigated.

Tile spacings of 40, 80, 160, and 320 feet were selected in order that the common spacings used in practice (25) could be bracketed. Hydraulic conductivities of 1, 2, 4, and 8 feet per day were believed to cover the range of soil conditions for the central area of Iowa (32). The value for drainable porosity was determined by using the relationship between hydraulic conductivity and drainable porosity as presented by the Bureau of Reclamation (18). The 4-foot depth to the impermeable layer was considered to be below the critical depth which would have an appreciable effect on water table prediction (44, 60). However, the condition for which the impervious layer was 0.5 foot below the tile was used to compare the two cases which had a common hydraulic conductivity of two feet per day.

There were times when a low hydraulic conductivity and a wide tile spacing caused the water table to accumulate above the ground surface. It was assumed that all ponded water should be removed as additional surface runoff. Therefore, the computations were programmed such that when the water table reached the ground surface, the remaining excess for that day was deleted.

Table 6. Summary of treatment conditions used to calculate the water table behavior from 1933 to 1962.

Tile Spacing ft.	Depth to impervious layer, h ft.	Hydraulic conductivity, K ft/day	Drainable porosity, f %	F	A days	A/f days
40	4.0	1.0	6.0	1.768	3.39	56.50
80	4.0	1.0	6.0	3.018	11.58	193.00
160	4.0	1.0	6.0	5.511	42.32	705.33
320	4.0	1.0	6.0	10.453	160.56	2670.00
40	4.0	2.0	10.0	1.768	2.83	28.30
80	4.0	2.0	10.0	3.018	9.66	96.60
160	4.0	2.0	10.0	5.511	35.27	352.70
320	4.0	2.0	10.0	10.453	133.79	1337.90
40	4.0	4.0	13.0	1.768	1.84	17.69
80	4.0	4.0	13.0	3.018	6.28	60.38
160	4.0	4.0	13.0	5.511	22.92	204.04
320	4.0	4.0	13.0	10.453	86.96	836.23
40	4.0	8.0	18.0	1.768	1.27	7.05
80	4.0	8.0	18.0	3.018	4.35	24.17
160	4.0	8.0	18.0	5.511	15.88	88.22
320	4.0	8.0	18.0	10.453	60.20	334.44
40	0.5	2.0	10.0	9.771	15.63	156.30
80	0.5	2.0	10.0	19.555	62.58	625.80
160	0.5	2.0	10.0	38.953	249.30	2493.00
320	0.5	2.0	10.0	77.550	992.64	9926.40

RESULTS AND DISCUSSION

Data Obtained from Water Balance

The precipitation for the months of April, May, and June over the period 1933-1962, and the surface runoff and excess as determined by the water balance procedure are presented in Table 7. The average precipitation for the three-month interval over the specified period was 11.60 inches.

Table 7. Precipitation, surface runoff and excess for April, May, and June from 1933 through 1962, Ames, Iowa

Year	Precipitation in.	Surface runoff in.	Excess in.
1933	5.01	1.36	1.44
1934	2.13	0.00	0.00
1935	13.61	2.04	4.57
1936	6.49	0.58	0.99
1937	9.82	0.28	1.95
1938	15.82	1.40	2.71
1939	8.09	0.24	0.00
1940	9.63	0.74	0.00
1941	13.47	2.34	3.63
1942	13.33	0.78	5.27
1943	10.89	0.70	4.47
1944	21.15	6.42	6.49
1945	13.41	1.06	3.87
1946	13.83	2.16	3.57
1947	23.32	4.58	10.11
1948	7.96	0.52	1.15
1949	6.41	0.00	0.00
1950	16.09	2.50	3.97
1951	15.68	2.06	3.28
1952	10.36	0.76	1.32
1953	9.62	0.58	0.95
1954	14.99	1.88	2.84
1955	10.36	0.70	2.06
1956	6.95	0.12	0.00
1957	15.29	2.32	4.33
1958	7.46	0.28	2.23
1959	13.28	1.34	3.32

Table 7 (Continued)

Year	Precipitation in.	Surface runoff in.	Excess in.
1960	16.37	3.22	4.73
1961	6.19	0.82	1.14
1962	10.89	0.84	3.50
Avg	11.60	1.42	2.80

This was about 38 percent of the average annual precipitation. The average surface runoff was 1.42 inches. In 1934 and 1939, no surface runoff occurred, as calculated. The maximum calculated surface runoff was 6.42 inches which occurred in 1944.

The average amount of excess moisture was 2.80 inches. There were five years with no calculated excess; the maximum excess was 10.11 inches, which occurred in 1947. This corresponded to the year of maximum rainfall which was 23.32 inches. However, the maximum runoff occurred in 1944 when there was a rainfall of 21.15 inches, and the excess was 6.49 inches. This situation was due to rainfall intensity and soil moisture conditions at the time of rainfall. It can be observed in Table 3 that the initial moisture profile was at field capacity 21 of the 30 years.

The assumption that the critical drainage period occurred during the season of April, May, and June may be evaluated by observing the contents of Table 8. The month of July had excess moisture about 25 percent of the time. However, the values did not exceed one inch except for the years of 1951 and 1958. The second most severe conditions occurred in October. This could be critical from the standpoint of harvesting. There were only five years when excess occurred during the period September-November. Since the amount of excess over the three-month intervals are comparable, it can be concluded that a sufficient drainage design for spring conditions should satisfy the fall drainage requirements.

Table 8. Excess moisture in inches for July, August, September, October, and November from 1933 through 1962, Ames, Iowa

Year	July	August	Sept.	Oct.	Nov.
1935					1.20
1938	.17				
1941	.43		0.32	4.56	1.16
1943	.02	1.43			
1951	1.04			1.22	0.57
1952	0.93				
1954				2.14	
1958	2.39				
1960	0.06				
1961			0.91	1.98	

Calculated Water Tables From 1933 Through 1962

The 20 treatment combinations previously defined (see Table 6) with the exception of the 40-foot spacing with a hydraulic conductivity of 8 feet per day, were used to calculate daily water table heights. This was accomplished by using Equation 21. The respective values for daily excess were substituted for P_N, where N took on consecutive values from 1 to 91. It was possible to eliminate the five years which had no calculated excess.

Although runoff was determined as a part of the water balance calculations, for certain conditions, such as a low hydraulic conductivity and a wide tile spacing, the water table exceeded the four-foot limit from the tile line to the soil surface. Two options were apparent: (1) it could be assumed that the remaining excess would be stored on the soil surface until complete infiltration of the stored excess occurred; or (2) it could be assumed that the remaining excess was removed as additional runoff. The latter assumption was used. When the excess for a given day caused the water table to rise above the ground surface, the excess necessary to bring the water table to the soil surface was calculated and subtracted from the total excess for that day. The remaining portion of the excess was recorded as additional runoff. These results are presented in Table 9. Six of the 20 treatment combinations did not remove the excess water at a rate sufficient to prevent surface accumulation. The most severe cases were for a value of K equal to one foot per day, h equal to four feet, and tile spacing of 160 feet or more. When K was two feet per day, and h was four feet, the only occurrence of additional runoff was for the 320-foot spacing, which occurred in 1944 and 1947. There were several instances of additional runoff when h was reduced to 0.5 foot,

Table 9. Portion of excess, designated as additional runoff, in inches, for April, May, and June, from 1933 through 1962

	Treatment combinations affected, where S = ft., K = ft./day, and h = ft.					
Year	h = 4 K = 1 S = 160	h = 4 K = 1 S = 320	h = 4 K = 2 S = 320	h = 0.5 K = 2 S = 80	h = 0.5 K = 2 S = 160	h = 0.5 K = 2 S = 320
1935	0.50	1.30				
1941	0.28	0.56				
1942	0.21	1.65				0.33
1943		0.86				
1944	1.35	2.93	0.70		1.12	1.55
1945		0.37				
1946		0.48				
1947	4.15	6.22	3.74	2.55	4.47	5.13
1950		0.77				
1957	0.77	1.23				
1959	0.09	0.34				
1960	0.72	1.82				0.28
1962	0.18	0.49				

and K was equal to two feet per day.

Frequency Distribution of Calculated Water Tables

Daily water table heights were calculated for 19 of the 20 treatment combinations for the months of April, May, and June for each year from 1933 through 1962. It was anticipated that the calculated water table fluctuations could be analyzed such that the average occurrence of a given event could be predicted.

The computational procedure was arranged to provide for the sorting and tabulation of water table heights for a given duration. This compilation was made for each treatment combination after the calculations were completed for the entire 30-year period. A sample of this sorting procedure is given in Table 10. The values in this table represent the number of times that the water table height was within a given range (the stated level up to 0.25 feet higher) for the number of days indicated during the 30-year period.

The next two steps were to cumulate the values in the rows of Table 10 from the right to the left, and then to cumulate the columns from the bottom to the top, by using the previously accumulated row values. An example of these steps is presented in Table 11, in which the data from Table 10 were used. The values in Table 11 represent the number of times during the 30-year period that a water table was as high or higher than that indicated by the column headings, and in interval of time as long or longer, than that indicated by the duration. For instance, there were 16 occasions when the water table was one foot or higher, above the tile for a duration of at least six days.

The data in the columns of Table 11 were treated in a manner similar to that which Langbein (36b) used to describe the expectancy of a flood. The partial-duration series was used, which is based upon all events above a selected base without regard to the number of events within any given time period. This method requires that the events be numbered with respect to size, beginning with the highest as number one.

Langbein stated that the base was generally selected to equal the lowest maximum annual event so that at least one event in each year could be included. In the partial-duration series, a recurrence interval was determined, which was defined as the average interval of time occurring between events of a size equaling, or exceeding, the selected size without regard to its relations to any period of time. The recurrence interval was computed using the formula,

$$RI = \frac{N + 1}{M}$$

where

RI = recurrence interval,

N = the number of years, and

M = the order of magnitude of events, beginning with the highest.

It was recognized that all events above the selected base, which occurred during the period of time described by the recurrence interval, should be used. Since a recurrence interval on a yearly basis was desired, it was necessary to assume that all water tables above a height of 0.75 foot occurred during the period of April, May, and June of each year. Although this assumption is not entirely correct, it can serve to describe the conditions for this limited period, which includes the interval of time when the need for drainage is greatest. It will be necessary to limit the use of the results to the same season from which the yearly recurrence

Table 10. Sorting and tabulation of water table heights for specified lengths of time using a tile spacing of 80 feet, a K-value of 2 feet per day and a depth of 4 feet below the tile to the impervious layer for April, May, and June from 1933 through 1962, Ames, Iowa

Duration	Water table height, feet									
days	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
1	5	7	4	5	5	2	0	1		
2	7	6	2	3	0	0	1			
3	6	5	3	2	1	1				
4	5	0	2	2	2					
5	4	4	3	0						
6	1	1	2	0						
7	0	3	1	1						
8	3	1	0	1						
9	0	2	0							
10	5	2	0							
11	1	0	0							
12	1	0	0							
13	1	0	0							
14	1	0	0							
15	0	0	0							
16	0	0	0							
17	0	0	0							
18	1	0	0							
19	0	1	1							
20	0									
.										
.										
.										
29	0									
30	1									

Table 11. Data from Table 10 in accumulated form

Minimum duration days	Minimum water table height, feet									
	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
1	119	77	45	27	13	5	2	1		
2	90	53	28	14	5	2	1			
3	71	41	22	10	4	1				
4	53	29	15	6	2					
5	42	23	9	2						
6	31	16	6	2						
7	27	13	4	2						
8	22	8	2	1						
9	17	6	1							
10	15	4	1							
11	8	2	1							
12	7	2	1							
13	6	2	1							
14	5	2	1							
15	4	2	1							
16	4	2	1							
17	4	2	1							
18	3	2	1							
19	1	2	1							
20	1									
.										
.										
.										
29	1									
30	1									

intervals were derived.

The event was defined as a minimum water table height for a minimum duration. The magnitude of the event was assumed to be directly related to the duration for a specific column of data in Table 11. Since the columns were cumulated from the bottom to the top, this provided an order of the magnitude. The base event was defined as the water table height indicated by the column heading and a duration of one day. Then, by using Equation 30, M could be determined for various recurrence intervals. This was accomplished for each column of Table 11. The results are presented in Figure 19, where recurrence intervals of 10, 5, 1, 1/2, and 1/3 years were used to determine the frequency series for the different water table heights. This information shows that a minimum water table height of 1.75 feet for a duration of 2 days or more, can be expected to have a recurrence interval of about 5 years. Likewise, a water table of 1 foot for a duration of 1.75 days can be expected to have a recurrence interval of 0.5 year.

The information presented in Figure 19 can also be presented by interchanging the position of recurrence interval and water table height. This has been accomplished in Figure 20, where recurrence intervals for 10, 5, 2, and 1 years are given as related to minimum water table heights and minimum durations.

Figure 19. Recurrence intervals of minimum water table heights for minimum durations based on the partial-duration series for a tile spacing of 80 feet, a K-value of 2 feet per day, and an h-value of 4 feet for April, May, and June from 1933 through 1962, Ames, Iowa

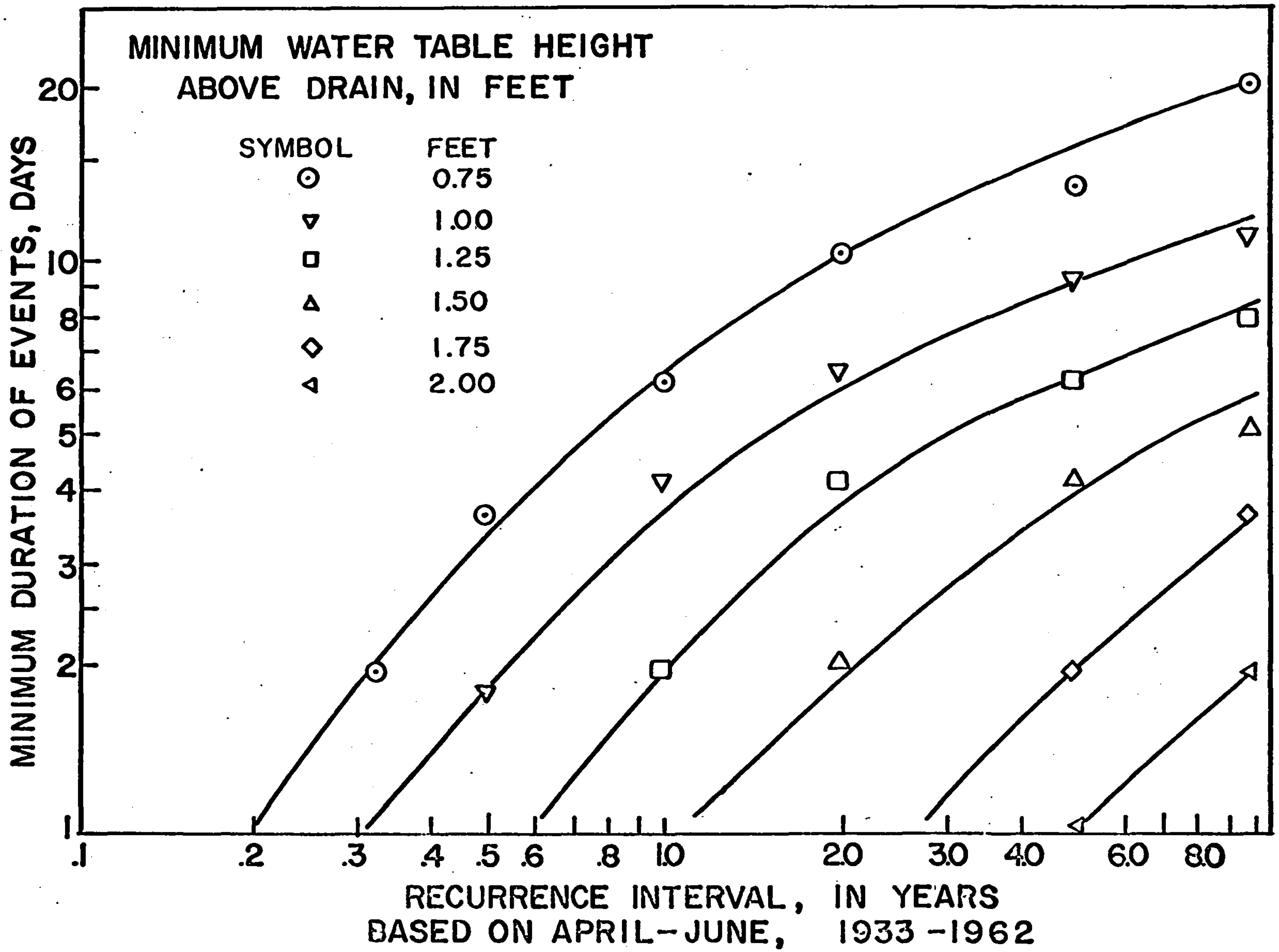
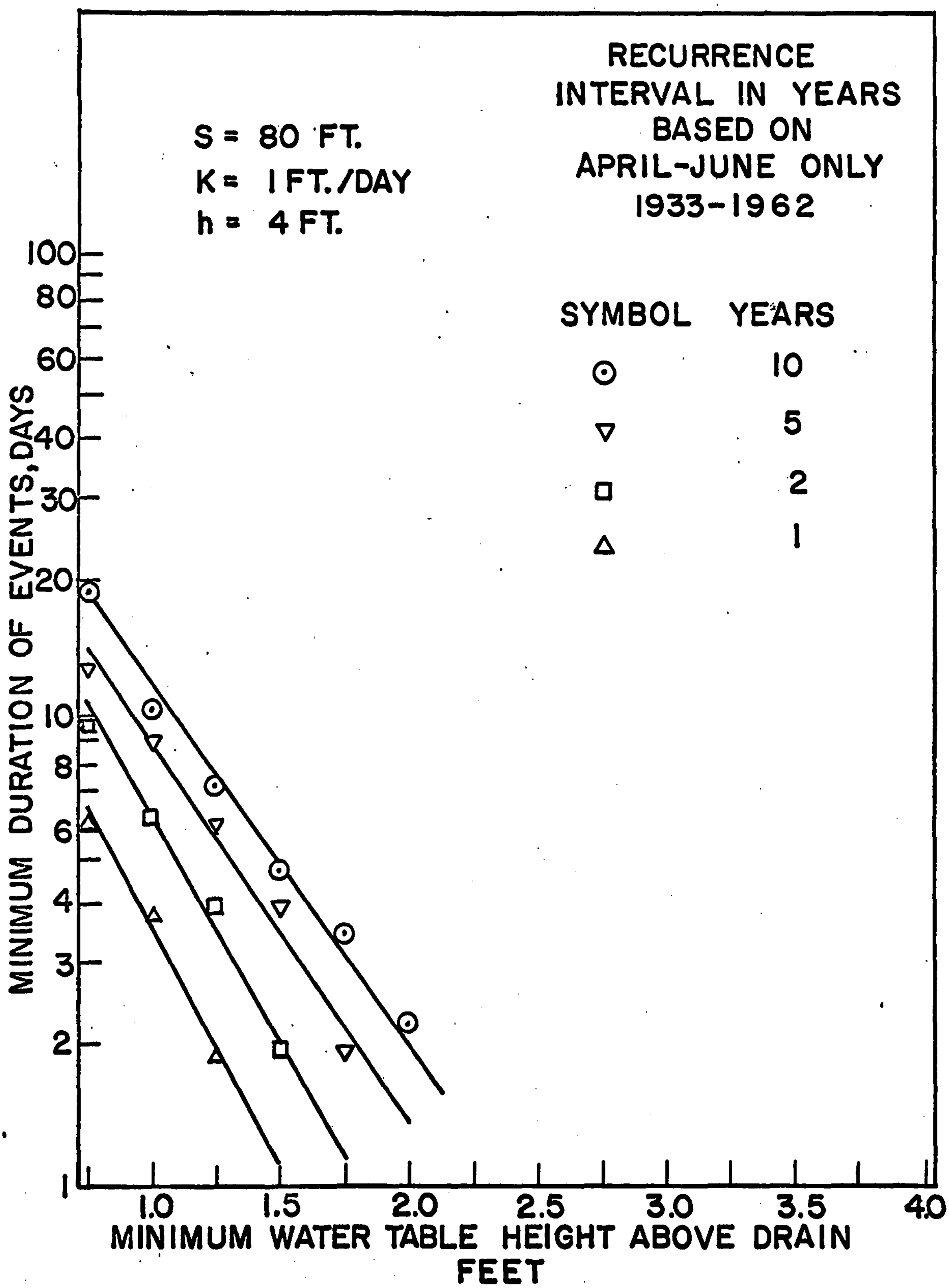

MINIMUM WATER TABLE HEIGHT
ABOVE DRAIN, IN FEET
SYMBOL
FEET
0.75
1.00
1.25
1.50
1.75
2.00
MINIMUM DURATION OF EVENTS, DAYS
20
10
8
6
5
4
3
2
1
.1
.2
.3
.4
.5
.6
.8
1.0
2.0
3.0
4.0
6.0
8.0
RECURRENCE INTERVAL, IN YEARS
BASED ON APRIL-JUNE, 1933-1962

Figure 20. Same as Figure 19 except that recurrence intervals and water table heights have been interchanged in positions on the graph


RECURRENCE INTERVAL IN YEARS BASED ON APRIL-JUNE ONLY 1933-1962
S = 80 FT.
K = 1 FT./DAY
h = 4 FT.
SYMBOL YEARS
10
5
2
1
MINIMUM DURATION OF EVENTS, DAYS
100
80
60
40
30
20
10
8
6
4
3
2
1
1.0
1.5
2.0
2.5
3.0
3.5
4.0
MINIMUM WATER TABLE HEIGHT ABOVE DRAIN FEET

The above procedure was used to present the results obtained from the various treatment combinations. The graphs in Figures 21, 22, 23, and 24 are for hydraulic conductivities of 1, 2, 4, and 8 feet per day respectively, with a 4-foot depth to the impervious layer in all cases. The graphs in Figure 25 are for a hydraulic conductivity of 2 feet per day, but the depth to the impervious layer was only 0.50 foot below the tile.

Recurrence intervals of 10, 5, 2, 1, 1/2, and 1/3 years were used where feasible. Since the water table did not exceed the 1-foot height for a 40-foot spacing when the value of K was 4 feet per day, and the value of h was 4 feet, the corresponding treatment for K equal to 8 feet per day was not calculated. Therefore, graphs for neither of these treatments were included. Also, since there were only 6 events for the 80-foot spacing for K equal to 8 feet per day, and h equal to 4 feet, the graph for this treatment was not included.

Smooth curves were drawn visually to the best fit of the points on the graphs. By placing all graphs for common K-values and h-values within the same figure, it is convenient to compare the effect of tile spacing upon the expected recurrence of a given drainage requirement.

It can be seen by observing Figure 21 (K=1, h=4) that the 40-foot spacing was adequate for almost any condition

Figure 21. Frequency of minimum water table heights for minimum durations when K = 1 foot per day, and h = 4 feet, Ames, Iowa

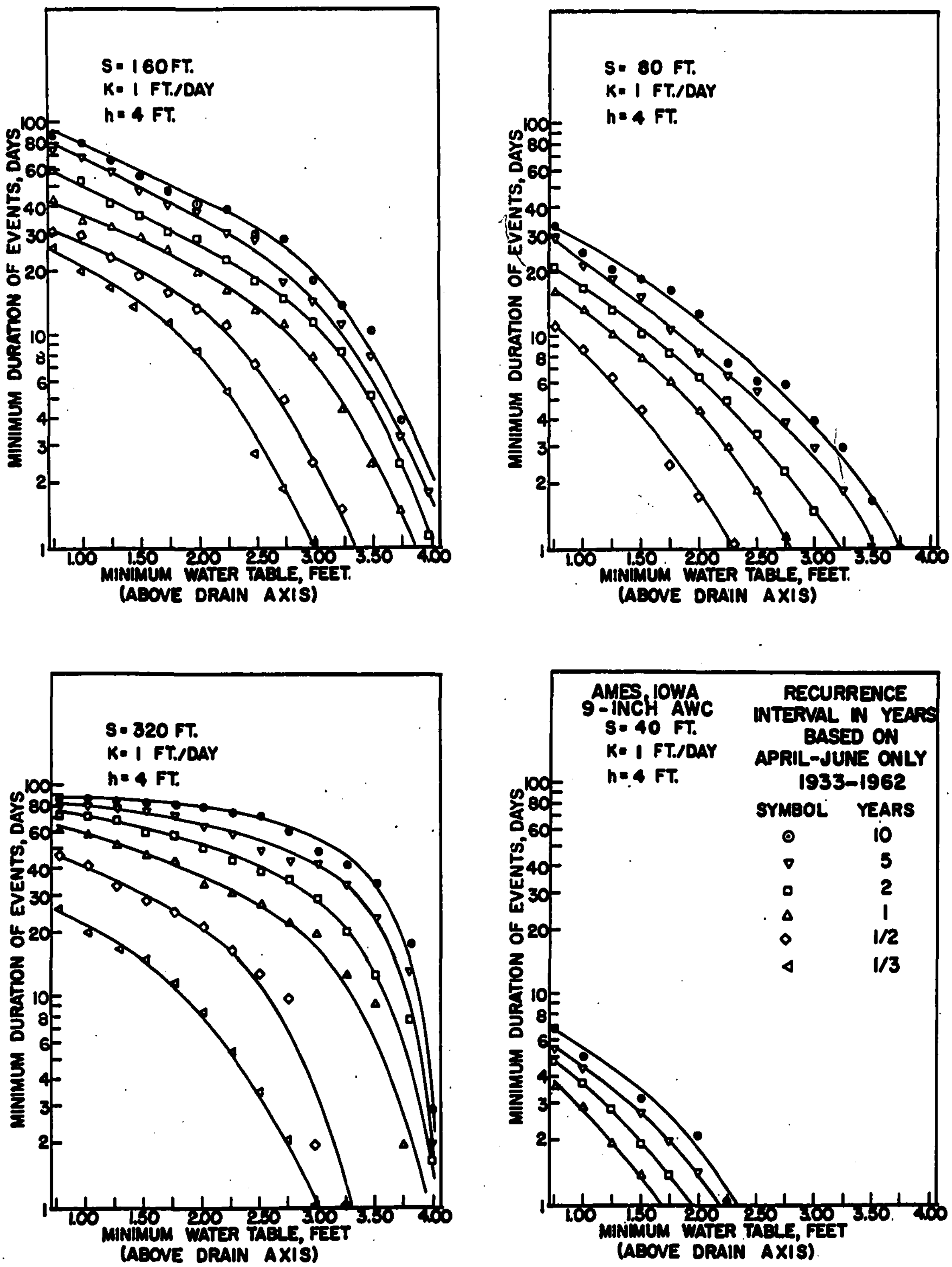

S= 160 FT.
K= 1 FT./DAY
h= 4 FT.
S= 80 FT.
K= 1 FT./DAY
h= 4 FT.
S= 320 FT.
K= 1 FT./DAY
h= 4 FT.
AMES, IOWA
9-INCH AWC
S= 40 FT.
K= 1 FT./DAY
h= 4 FT.
RECURRENCE INTERVAL IN YEARS BASED ON APRIL-JUNE ONLY 1933-1962
SYMBOL YEARS
10
5
2
1
1/2
1/3
MINIMUM DURATION OF EVENTS, DAYS
MINIMUM WATER TABLE, FEET (ABOVE DRAIN AXIS)
1.00 1.50 2.00 2.50 3.00 3.50 4.00
1 2 3 4 6 8 10 20 30 40 60 80 100

Figure 22. Frequency of minimum water table heights for minimum durations when K = 2 feet per day and h = 4 feet, Ames, Iowa

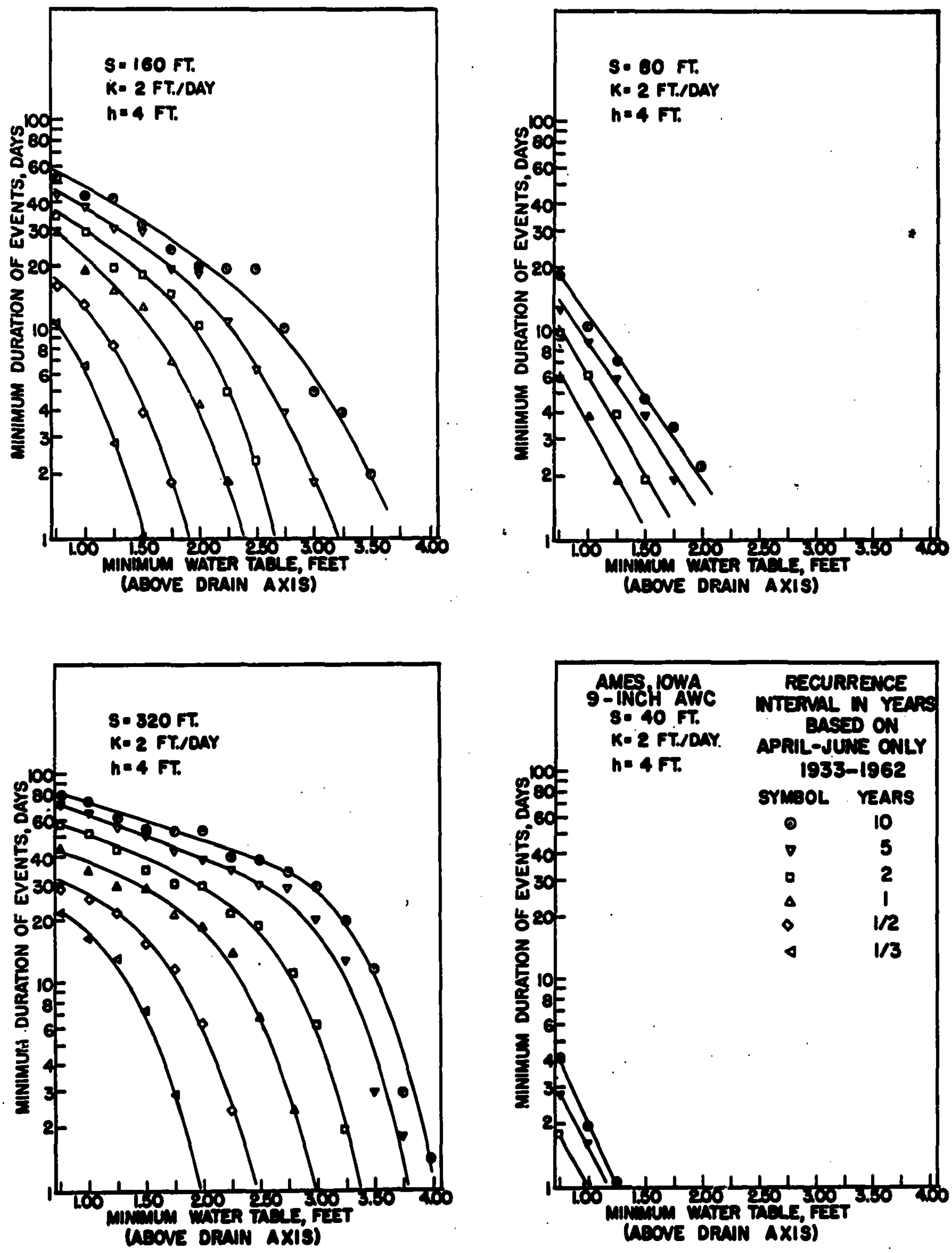

S = 160 FT.
K = 2 FT./DAY
h = 4 FT.
S = 80 FT.
K = 2 FT./DAY
h = 4 FT.
S = 320 FT.
K = 2 FT./DAY
h = 4 FT.
AMES, IOWA
9-INCH AWC
S = 40 FT.
K = 2 FT./DAY
h = 4 FT.
RECURRENCE INTERVAL IN YEARS BASED ON APRIL-JUNE ONLY 1933-1962
SYMBOL YEARS
10
5
2
1
1/2
1/3
MINIMUM DURATION OF EVENTS, DAYS
MINIMUM WATER TABLE, FEET (ABOVE DRAIN AXIS)

Figure 23. Frequency of minimum water table heights for minimum durations when $K = 4$ feet per day and $h = 4$ feet, Ames, Iowa

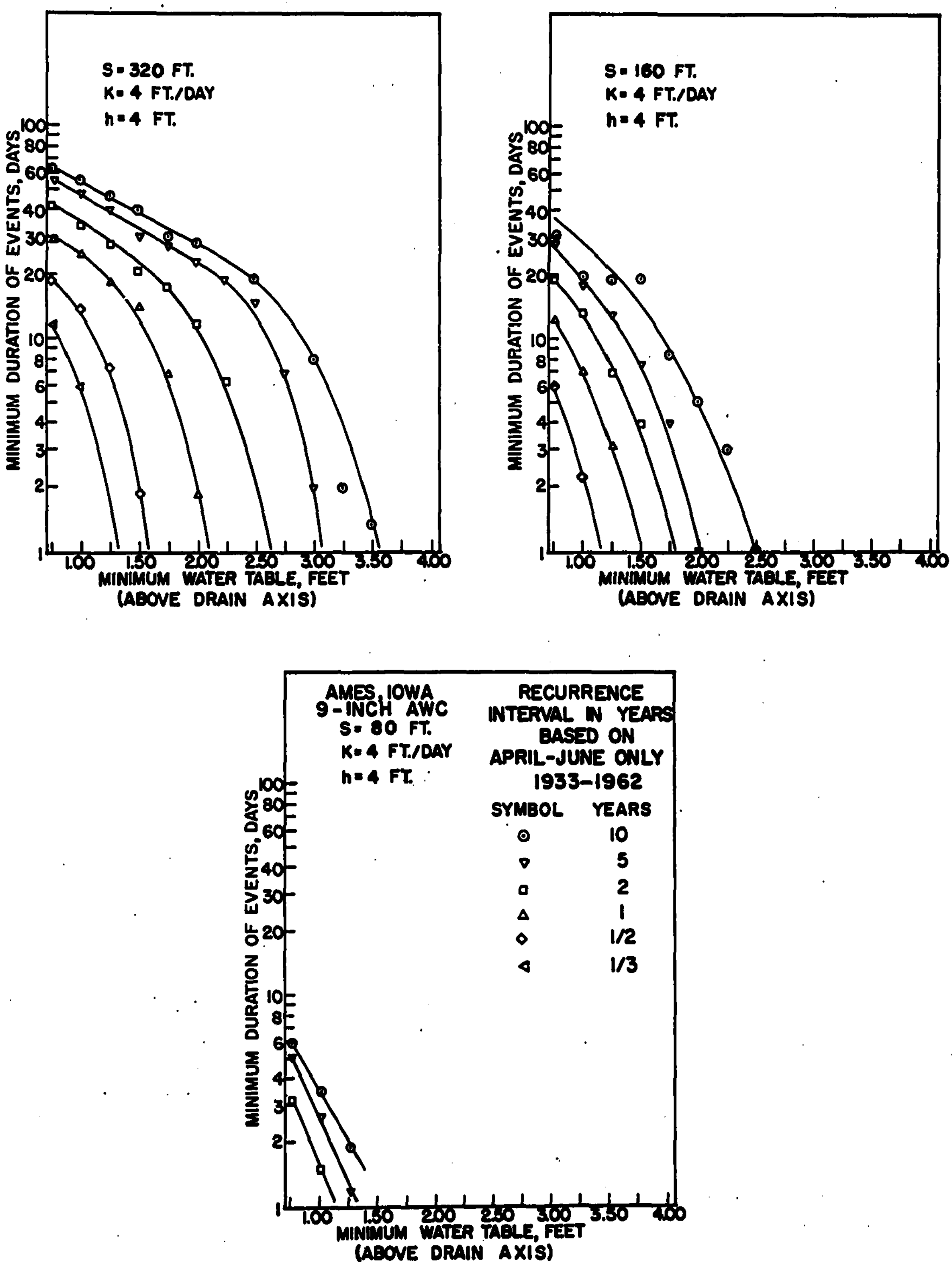
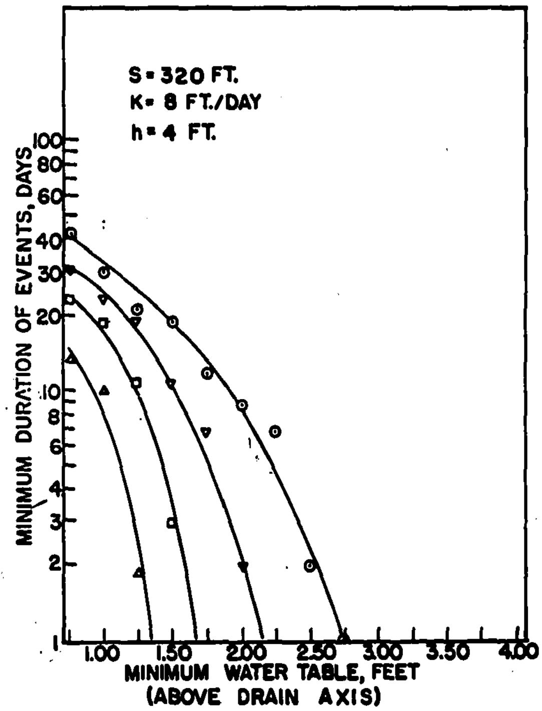

S = 320 FT.
K = 4 FT./DAY
h = 4 FT.
S = 160 FT.
K = 4 FT./DAY
h = 4 FT.
AMES, IOWA
9-INCH AWC
S = 80 FT.
K = 4 FT./DAY
h = 4 FT.
RECURRENCE INTERVAL IN YEARS BASED ON APRIL-JUNE ONLY 1933-1962
SYMBOL YEARS
10
5
2
1
1/2
1/3
MINIMUM DURATION OF EVENTS, DAYS
MINIMUM WATER TABLE, FEET (ABOVE DRAIN AXIS)

Figure 24. Frequency of minimum water table heights for minimum durations when K = 8 feet per day and h = 4 feet, Ames, Iowa


S = 320 FT.
K = 8 FT./DAY
h = 4 FT.
MINIMUM DURATION OF EVENTS, DAYS
MINIMUM WATER TABLE, FEET
(ABOVE DRAIN AXIS)

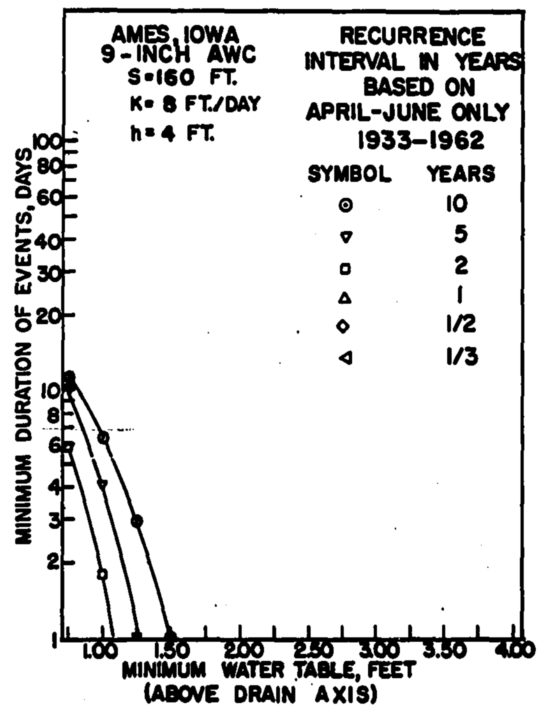
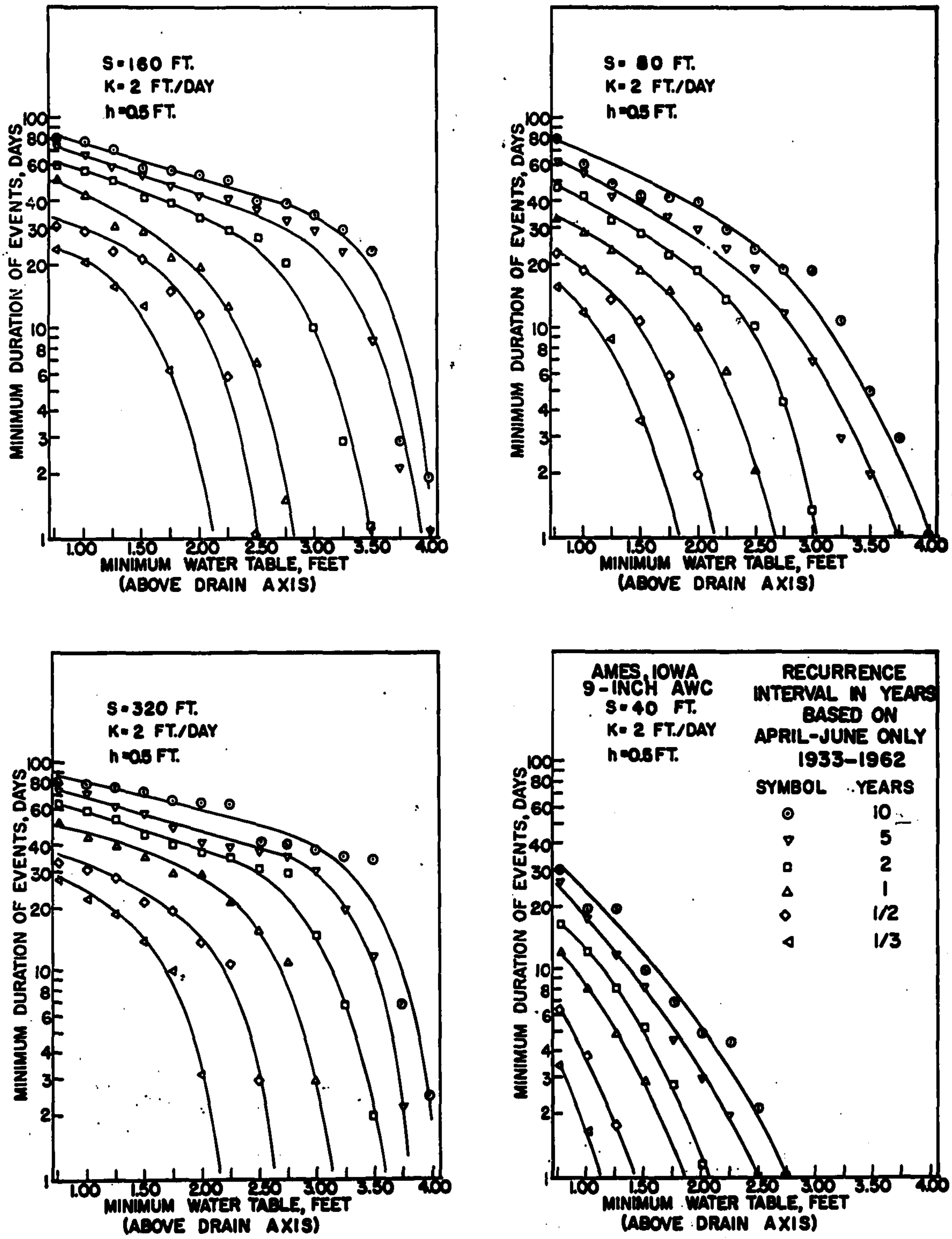

AMES, IOWA
9-INCH AWC
S = 160 FT.
K = 8 FT./DAY
h = 4 FT.
RECURRENCE INTERVAL IN YEARS BASED ON APRIL-JUNE ONLY 1933-1962
SYMBOL YEARS
10
5
2
1
1/2
1/3
MINIMUM DURATION OF EVENTS, DAYS
MINIMUM WATER TABLE, FEET
(ABOVE DRAIN AXIS)

Figure 25. Frequency of water table heights for minimum durations when K = 2 feet per day and h = 0.5 foot, Ames, Iowa


S=160 FT.
K=2 FT./DAY
h=0.5 FT.
S=80 FT.
K=2 FT./DAY
h=0.5 FT.
S=320 FT.
K=2 FT./DAY
h=0.5 FT.
AMES, IOWA
9-INCH AWC
S=40 FT.
K=2 FT./DAY
h=0.5 FT.
RECURRENCE INTERVAL IN YEARS BASED ON APRIL-JUNE ONLY 1933-1962
SYMBOL YEARS
10
5
2
1
1/2
1/3
MINIMUM DURATION OF EVENTS, DAYS
MINIMUM WATER TABLE, FEET (ABOVE DRAIN AXIS)

encountered in practice. A minimum water table of two feet above the tile would be expected to last for a minimum of two days with a 10-year recurrence interval. The spacings larger than 40 feet, for the given K and h values, offer low protection. The maximum allowable spacing would be between 40 and 80 feet.

The conditions shown in Figure 22 (K=2, h=4) indicate that 80-foot tile spacings would satisfy most drainage demands in practice, where comparable soil conditions prevailed. A minimum water table of 2 feet for a duration of 2 days or more, would be expected for only 1 year in 10. The 160-foot spacing would permit a 3-foot water table for a period of 6 days or more, with a 10 year recurrence interval. The accepted spacing for comparable soil conditions would be between 80 and 160 feet.

The case presented in Figure 23 (K=4, h=4) indicates that a tile spacing of 160 feet would allow a minimum water table of 2 feet for a duration of 5 days or more, only once in 10 years, on the average. This protection would be considered excellent. When the tile spacing is increased to 320 feet, a 3-foot or higher water table with a duration of 2 days or more, would be expected with a 5-year recurrence interval. The allowable spacing would be between 160 and 320 feet.

The results shown in Figure 24 (K=8, h=4) suggest that tile spacings greater than 320 feet should have been considered, since a water table of 2.75 feet or higher, for 1 day or more, would be expected only 1 time in 10 years.

An impervious layer near the tile (h=0.5 ft.) considerably increased the chance of a high water table. This can be observed by comparing the graphs in Figure 25 to those in Figure 22. The value of K was two feet per day in both cases, the only difference being the location of the impervious layer. The greatest effect of the reduced value of h was at the close spacing, with the effect gradually decreasing until it could barely be detected at the 320-foot spacing. The 80-foot spacing with a 4-foot value of h offered better protection than the 40-foot spacing where the value of h was 0.5 foot. The same situation existed between the 160-foot spacing, where h was equal to 4 feet, and the 80-foot spacing for h equal to 0.5 foot.

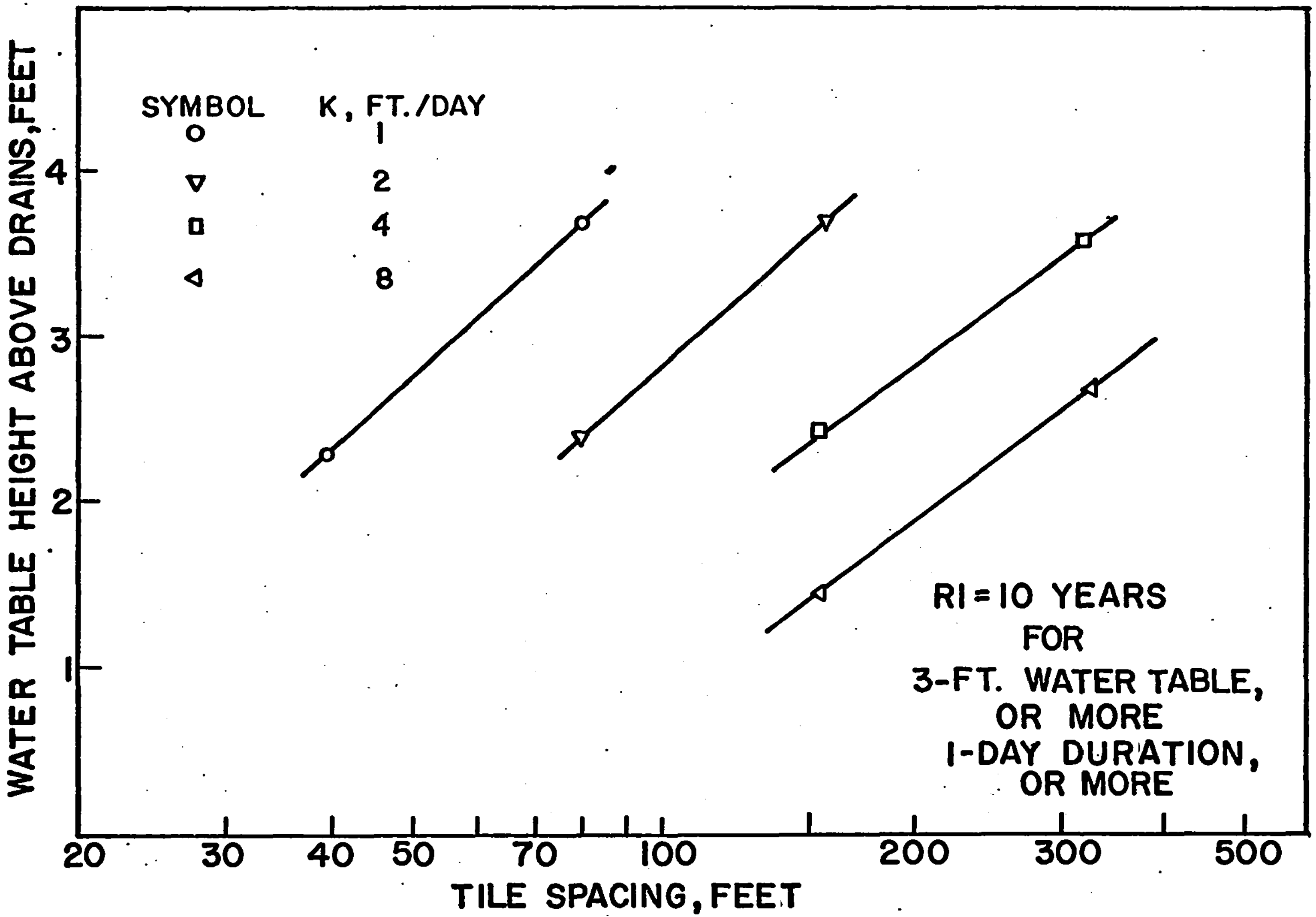
Tile spacings for given values of K, h, and recurrence intervals may be interpolated from the frequency distributions. For example, suppose that the required condition is that the water table will be limited to a height of three feet or more, for a period of one day or more. The tile will be four feet deep, and the impervious layer will be four feet below the tile. Semi-logarithmatic paper can be used to plot tile spacing on the logarithmic scale, and height of water table on the linear scale, as shown in Figure 26. Then it is necessary to bracket the three-foot water table heights by selecting from the frequency curves the respective values for the conditions required. The values of water table height for K equal to one foot per day were 2.35 feet and 3.75 feet from Figure 21 for tile spacings of 40 and 80 feet, respectively. These values were plotted on the graph in Figure 26. The points were connected with a straight line; it was found that the three-foot level was intersected at a spacing of 55 feet.

A similar procedure was followed for the other values of K. Tile spacings of 110, 215, and 385 feet were determined when the values of K were 2, 4, and 8 feet respectively, based on a water table height of three feet or more, for a minimum duration of one day, and a recurrence interval of 10 years. The frequency distributions could be utilized in a similar fashion for a number of drainage conditions. The values interpolated between the points plotted should have reasonable accuracy, however extrapolation beyond the points may lead to serious error.

The Relationship of Drainage Coefficients to Calculated Water Tables

The previous investigations were based on the condition which assumed tile flowing almost full, therefore it was desirable to determine whether there were times when this flow

Figure 26. Procedure for interpolating tile spacings from frequency distributions for given drainage conditions



SYMBOL K, FT./DAY
1
2
4
8
RI = 10 YEARS
FOR
3-FT. WATER TABLE,
OR MORE
1-DAY DURATION,
OR MORE
WATER TABLE HEIGHT ABOVE DRAINS, FEET
4
3
2
1
20 30 40 50 70 100 200 300 500
TILE SPACING, FEET

condition was exceeded. Drainage coefficients of 3/8 inch and 1/4 inch were used in conjunction with the graphical solution given by Toksoz and Kirkham (63). The height of the water table was determined, which gave steady-state conditions for precipitation comparable to the drainage coefficient. These values were compared to the calculated water tables for the various values of K, h, and S. The results of treatments which were affected are given in Table 12. The maximum water table heights calculated over the 30-year period can be observed approximately in Figures 21 through 25 by noting where the curve for the 10 year recurrence interval intersects the abscissa.

If a 3/8-inch drainage coefficient was used in design, no cases were found where the tile flow restricted drainage. However, if conditions related to the 1/4-inch drainage coefficient are studied, it is found that the critical condition was reached several times. When the conditions were K = 1, h = 4, and S = 40, (Figure 21) the critical water table height of 1.55 feet was exceeded about once each year on the average, when the duration was one day or more.

It should be noted that the frequency distributions do not apply when the critical water table heights are exceeded. When these circumstances prevailed, the actual water table was higher than that calculated, and the tile mains would be flowing full and under pressure.

The critical value of the water table height for K=2, h=4, and S=80 was 0.75 feet, compared to a maximum calculated value of 1.25 feet. On the average, the critical value was exceeded about every other year for a duration of at least one day (see Figure 22). For the conditions K=4, h=4, and S=80, the critical value of the water table height was 1.25 feet, compared to a maximum calculated value of 1.50 feet. A recurrence interval of about five years can be observed from Figure 23 for this condition.

Table 12. Water table heights necessary to produce outlet discharge comparable to respective drainage coefficients and tile spacings

Treatment	Drainage coefficient 3/8 inch Height of water table ft.	Drainage coefficient 1/4 inch Height of water table ft.
K = 1		
h = 4		
S = 40	2.25	1.55
S = 80	7.30	4.90
K = 2		
h = 4		
S = 40	1.10	0.75
S = 80	3.80	2.50
K = 4		
h = 4		
S = 40	0.55	0.39
S = 80	1.95	1.25
S = 160	6.20	4.60
K = 8		
h = 4		
S = 40		0.20
S = 80	0.95	0.60
S = 160	3.10	2.30

Table 12 (Continued)

	Drainage coefficient 3/8 inch Height of water table ft.	Drainage coefficient 1/4 inch Height of water table ft.
K = 2		
h = 0.5		
S = 40	6.5	4.25

It may be concluded that tile systems designed on a 3/8 inch drainage coefficient will not flow under pressure for the conditions used or assumed in this study, while design on a 1/4-inch drainage coefficient would not be sufficient to prevent pressure in the tile system.

There would be exceptions associated with the assumption of surface runoff, even for the 3/8-inch coefficient, in cases where water could be stored on the surface. These conditions have been encountered in practice[3]. However, it may be pointed out that after ponded conditions occur, the flow through the soil above the tile is increased more than the flow through the soil farther away from the tile. This, in one respect, is equivalent to surface inlets, which were not considered for the above drainage coefficients. Therefore, when ponded conditions are anticipated, a larger drainage coefficient must be used if the condition of pressure in the tile system is to be reduced or eliminated.

[3]Data from files of Agricultural Engineering Department, Iowa State University, Ames, Iowa.

SUMMARY AND CONCLUSIONS

The purpose of this study was to develop a design procedure for the spacing of tile drains in agricultural soils. The information was developed for central Iowa rainfall patterns under continuous corn cropping. Information on the expected frequency of a given degree of drainage protection was desired.

The above purpose was achieved by determining the proportion of precipitation that produced the need for drainage, and by relating this accumulation of excess moisture to the behavior of the water table for various tile spacings, soil conductivities, and locations of the impervious layer. It was assumed that all tile installations were made at a depth of four feet, and that an impervious layer below the tile prohibited deep percolation. It was also assumed that the primary demand for drainage could be limited to the period of April, May, and June.

The portion of rainfall that contributed to drainage needs was determined by the use of a water balance. The daily precipitation was divided into evaporation (including transpiration), runoff, and storage. The average rainfall for April, May, and June over the 30 years studied was 11.60 inches, slightly more than the long record average. The average surface runoff during the three months was 1.42 inches, and the average depth of water to be drained (termed excess), was 2.8 inches.

The excess for one three-month period was applied to a viscous fluid model which simulated a representative soil profile with tile drains installed. The same time sequence of rainfall was followed, which occurred naturally. The observed water table fluctuations were compared to the results of two analytical methods. The Kraijenhoff equation produced results continually higher than the observed model values, with extreme departures on the day which excess occurred. This lack of agreement was contributed to the procedure used in translating steady state conditions to transient conditions.

The van Schilfgaarde equation was consistently below the values observed in the model. The difference amounted to a factor which was about the same as the capillary fringe measured in the model.

It was concluded that the latter development gave more accurate results based on the comparison with the model. This equation was programed on an IBM 7074 computer, and daily water tables were calculated for hydraulic conductivities of 1, 2, 4, and 8 feet per day, tile spacings of 40, 80, 160, and 320 feet, and a depth to the impervious layer of 4 feet below the tile. Also, to compare the affect of a shallow impervious layer, the condition of a hydraulic conductivity of two feet per day and a depth to the impervious layer of 0.5 foot was applied to the four tile spacings.

Frequency distributions were developed from a tabulation of the calculated daily water tables. A recurrence interval was established for a water table equal to or greater than a given height, for a duration equal to or greater than the specified duration. The recurrence interval was based on water table data from April, May, and June for the years 1933 through 1962. Therefore, in applying the results, this limitation must be considered.

A shallow impervious layer located 0.5 foot below the tile required closer tile spacings to secure the same degree of protection as obtained when the impervious layer was four feet below the tile. The comparison was made with a hydraulic conductivity of two feet per day in both cases. The affect of the shallow impervious layer was not as great when the tile spacing exceeded 160 feet.

It was found that an outlet for the drainage system designed for a 3/8-inch drainage coefficient provided a capacity sufficient to prevent flow under pressure for all treatments used. However, when the drainage coefficient was reduced to 1/4 inch, it was found that the water table height exceeded the maximum allowed for flow without pressure in three different cases. The 40-foot spacing for K equal to 1 and 2 feet per day, and 80-foot spacing for K equal to 4 feet per day resulted in drainage coefficients larger than 1/4 inch. In each case, the depth to the impervious layer was located four feet below the drains.

The following conclusions were made:

1. The design of tile drainage systems by the use of water table frequencies based on soil characteristics, crops, and climatological data now appears feasible.
2. Drain tile spacings of 55, 110, 215, and 385 feet are specified for hydraulic conductivities of 1, 2, 4, and 8 feet per day respectively, under the following conditions:
   (a) Tile 4 feet deep, and an impervious layer 4 feet below the tile.
   (b) A water table height of three feet for a duration of one day (recurrence interval of 10 years).
3. A drainage coefficient of 1/4 inch is too small to avoid pressure in a tile system at peak drainage requirements. A drainage coefficient of 3/8 inch is adequate when there are no surface inlets, and ponded conditions do not exist.

The method of tile design developed in this study will be useful in making economic evaluations where the frequency of the reduction of crop yields due to poor drainage can be compared to the cost of drainage facilities. It may also be adapted to aid in the study of field systems where the expected arrival date for certain field operations is related to pertinent climatic factors, especially soil-moisture conditions.

Further studies along this line should extend such factors as depth of tile, the prediction of moisture use, the depth to the impervious layer, and the relationship of the hydraulic conductivity to the drainable porosity. The affect of installing tile at depths of 3 and 5 feet should be compared to the results of this investigation, where the depth was four feet. It is believed that the water balance procedure used gives a good account of the various fractions of rainfall. Additional investigations should be pursued in adapting the procedure to other crops, and other areas of the state. The depth to the impervious layer needs further investigation to see whether there is a depth at which drainage design could be subdivided into two categories, one where the impervious layer is above this depth and the other one below.

A definite relationship between hydraulic conductivity and drainable porosity was used in this investigation. Further work is needed to more adequately define how this relationship varies.

Such problems as stratification and variation in hydraulic conductivity continually plague the designer. The affect of these problems may be best evaluated through systems installed, or by installing a tile system where the variables can be characterized. Studies along this line are needed in order to adapt the present investigation to actual field conditions.

The problem of designing drainage facilities for small wet areas, caused by depressions of the surface, or by local sand pockets, will not be entirely solved by the procedure developed in this study. These areas will require additional drainage to remove the accumulated excess caused by runoff and seepage from surrounding areas.

LITERATURE CITED

1. Aronovinci, V. S. and W. W. Donnon. Soil permeability as a criterion for drainage design. American Geophysical Union Transactions 27: 95-101. 1946.

2. Asseed, M. S. Unsteady and steady flow of fluid in a glass bead glycerol land drainage model. Unpublished M.S. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1957.

3. Baver, L. D. Soil physics. 3rd ed. New York, New York, John Wiley and Sons, Inc. 1956.

4. Beauchamp, K. H. and G. B. Fasken. Drainage recommendations by soil types. Agricultural Engineering 36: 248-249. 1955.

5. Blake, G. R., E. R. Allred, C. H. M. van Bavel, and F. D. Whisler. Agricultural drought and moisture excesses in Minnesota. Minnesota Agricultural Experiment Station Technical Bulletin 235. 1960.

6. Bouwer, H. Theoretical aspects of flow above the water-table in tile drainage of shallow homogeneous soils. Soil Science Society of American Proceedings 23: 260-263. 1959.

7. Bouwer, H. and J. van Schilfgaarde. Simplified method of predicting fall of water table in drained land. American Society of Agricultural Engineers Transactions 6: 288-291, 296. 1963.

8. Buss, S. and R. H. Shaw. Prediction of soil moisture under corn. Part 2. Multilithed final report to U.S. Weather Bureau on Contract CWB-9560. Ames, Iowa, Department of Agronomy, Iowa State University of Science and Technology. 1960.

9. Cassell, W. L. Linear electric circuits. New York, New York, John Wiley and Sons, Inc. 1964.

10. Childs, E. C. The water table equipotentials and streamlines in drained land. V. The moving water table. Soil Science 63: 361-376. 1947.

11. Dale, R. F. Weather effects on experimental plot corn yields: climatology of selected favorable and unfavorable conditions. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1964.

12. Dale, R. F. and M. Hartley. Computer program for estimating soil moisture under corn. Multilithed appendix to final report to U.S. Weather Bureau on Contract CWB-10554. Ames, Iowa, Department of Agronomy, Iowa State University of Science and Technology. 1963.

13. Denmead, O. T. and R. H. Shaw. Availability of soil water to plants as affected by soil moisture conditions and meteorological conditions. Agronomy Journal 45: 385-390. 1962.

14. Denmead, O. T. and R. H. Shaw. Evapotranspiration in relation to the development of a corn crop. Agronomy Journal 51: 725-726. 1959.

15. Dumm, L. D. Drain-spacing formula. Agricultural Engineering 35: 726-730. 1954.

16. Dumm. L. D. Transient-flow concept in subsurface drainage: its validity and use. American Society of Agricultural Engineers Transactions 7: 142-146, 151. 1964.

17. Dumm, L. D. and R. J. Winger, Jr. Subsurface drainage system design for irrigated area using transient flow concept. American Society of Agricultural Engineers Transactions 7: 147-151. 1964.

18. Dylla, A. S. Simplified procedure for transient-flow drain design. Unpublished mimeographed paper presented at American Society of Agricultural Engineers meeting, New Orleans, Louisiana, December, 1964. Reno, Nevada, Agricultural Research Service, U.S. Department of Agriculture. ca. 1964.

19. Ede, A. N. Scheme for the assessment and design of field drainage systems on a hydrologic basis. Congress of Soil Science, 7th, Transactions 1: 493-505. 1960.

20. Frevert, R. K., G. O. Schwab, T. W. Edminster, and K. K. Barnes. Soil and water conservation engineering. New York, New York, John Wiley and Sons, Inc. 1955.

21. Grover, B. L. Non-steady state water tables of tile drainage. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1959.

22. Grover, B. L. and D. Kirkham. Solving tile drainage problems by using model data. Iowa Agricultural Experiment Station Research Bulletin 523. 1964.

23. Holl, L., C. G. Maple, and B. Vinograde. Introduction to the Laplace transform. New York, New York, Appleton-Century-Crofts, Inc. 1959.

24. Hooghoudt, S. B. Bijdragen tot de kennis van eenige natuurkundige grootheden van den grond, 7. Algemeene beschouwing van hat probleem van de detail onwatering en de infiltratie door midel van parallel loopende drains, greppels, slooten en kanalen. Verslagen, Landbouwkundige Onderzoekingen 46: 515-707. 1940. Original not available; cited in Schwab, G. O. Engineering aspects of land drainage. Agronomy 7: p. 386. 1957.

25. Iowa drainage guide. Iowa Agricultural Experiment Station Special Report 13 (revised). 1962.

26. Kirkham, D. Physical artifices and formulas for approximating water table fall in tile-drained land. Soil Science Society of America Proceedings 28: 585-590, 677. 1964.

27. Kirkham, D. Seepage of steady rainfall through soil into drains. American Geophysical Union Transactions 39: 892-907. 1958.

28. Kirkham, D. An upper limit for the height of the water table in drainage design formulas. Congress of Soil Science, 7th, Transactions 1: 486-492. 1960.

29. Kirkham, D. and R. E. Gaskell. The falling water table in tile and ditch drainage. Soil Science Society of America Proceedings 15: 37-42. 1950.

30. Kirkham, D. and J. W. De Zeeuw. Field measurements for tests of soil drainage theory. Soil Science Society of America Proceedings 16: 286-293. 1952.

31. Klinge, A. F. Tile drainage of a heavy soil. Unpublished M.S. thesis. Lafayette, Indiana, Library, Purdue University. 1955.

32. Klock, G. O. Horizontal and vertical hydraulic conductivity measurements of some Iowa soil types. Unpublished M.S. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1963.

33. Kohler, M. A. and R. R. Linsley. Predicting runoff from storm rainfall. U.S. Weather Bureau Research Paper 34. 1951.

34. Kraijenhoff, van de Leur, D. A. Some effects of the unsaturated zone on nonsteady free-surface groundwater flow as studied in a scaled granular model. Journal of Geophysical Research 67: 4347-4362. 1962.

35. Kraijenhoff, van de Leur, D. A. A study of non-steady groundwater flow. II. Computation methods for flow to drains. Ingenieur 74: B285-B292. 1962.

36a. Kraijenhoff, van de Leur, D. A. A study of non-steady groundwater flow with special reference to a reservoir-coefficient. Ingenieur 70: B87-B94. 1958.

36b. Langbein, W. B. Annual floods and the partial-duration series. American Geophysical Union Transactions 30: 879-881. 1949.

37. Ligon, J. T. Models of unsteady state flow in porous media applied to soil drainage. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology. 1961.

38. Ligon, J. T., G. R. Benoit, and A. B. Elan, Jr. A procedure for determining the probability of soil moisture deficiency and excess. Unpublished mimeographed paper presented at American Society of Agricultural Engineers meeting, Ft. Collins, Colorado, June, 1964. Lexington, Kentucky, Department of Agricultural Engineering, University of Kentucky. ca. 1964.

39. Ligon, J. T., H. P. Johnson, and K. Kirkham. Glass bead-clycerol model for studying falling water table between open ditches. American Society of Agricultural Engineers Transactions 6: 61-64. 1963.

40. Ligon, J. T., H. P. Johnson, and D. Kirkham. Unsteady-state drainage of fluid from a vertical column of porous material. Journal of Geophysical Research 67: 5199-5204. 1962.

41. Ligon, J. T., D. Kirkham, and H. P. Johnson. The falling water table between open ditch drains. Soil Science 97: 113-119. 1964.

42. Linsley, R. K., Jr., M. A. Kohler, and J. L. H. Paulhus. Applied hydrology. New York, New York, McGraw-Hill Book Company, Inc. 1949.

43. Luthin, J. N. The falling water table in tile drainage. II. Proposed criteria for spacing tile drains. American Society of Agricultural Engineers Transactions 2: 44-45. 1959.

44. Luthin, J. N. and R. V. Worstell. The falling water table in tile drainage. III. Factors affecting the rate of fall. American Society of Agricultural Engineers Transactions 2: 44-47, 51. 1959.

45. Lynde, H. M. Tile drainage investigations in North Carolina. Agricultural Engineering 2: 133-135. 1921.

46. Maasland, M. Water table fluctuations induced by intermittent recharge. Journal of Geophysical Research 64: 549-559. 1959.

47. Marlatt, W. E., H. V. Havens, N. A. Willets, and G. D. Brill. A comparison of computed and measured soil moisture under snap beans. Journal of Geophysical Research 66: 535-541. 1961.

48. Mason, D. D., J. F. Lutz, and R. G. Petersen. Hydraulic conductivity as related to certain soil properties in a number of great soil groups; sampling errors involved. Soil Science Society of America Proceedings 21: 554-560. 1957.

49. Muskat, M. The flow of homogeneous fluids through porous media. Ann Arbor, Michigan, J. W. Edwards, Inc. 1946.

50. Neal, J. H. Proper spacing and depth of tile drains determined by the physical properties of the soil. Minnesota Agricultural Experiment Station Technical Bulletin 101. 1934.

51. Peele, T. C. Relation of percolation rates through saturated cores to volume of pores drained in 15 and 30 minutes under 60 centimeters tension. Soil Science Society of America Proceedings 14: 359-361. 1949.

52. Penman, H. L. Natural evaporation from open water, bare soil and grass. Royal Society of London Proceedings Series A, 193: 120-145. 1948.

53. Russell, J. L. Scientific research in soil drainage. Journal of Agricultural Science 24: 544-573. 1934.

54. Sabersky, R. H. and A. J. Acosta. Fluid flow. New York, New York, The Macmillan Company. 1964.

55. Schlick, W. J. The spacing and depth of laterals in Iowa underdrainage systems and the rate of runoff from them with data from investigations. Iowa State College of Science and Technology Engineering Experiment Station Bulletin 52. 1918.

56. Schwab, G. O. Engineering aspects of land drainage. Agronomy 7: 385-394. 1957.

57. Shaw, R. H. Estimation of soil moisture under corn. Iowa Agricultural Experiment Station Research Bulletin 520. 1963.

58. Shaw, R. H. The prediction of soil moisture under corn and meadow for the winter period in Iowa. Multilithed final report to U.S. Weather Bureau on Contract CWB-10554. Ames, Iowa, Department of Agronomy, Iowa State University of Science and Technology. 1963.

59. Stol, P. T. A statistical analysis of the differences between precipitation and evaporation in The Netherlands. In Conference on Supplemental Irrigation Commission VI. International Society of Soil Science Report, 1958. pp. 41-49. Wageningen, The Netherlands, Institute for Land and Water Management Research. 1958.

60. Talsma, T. and H. C. Haskew. Investigation of water table response to drains in comparison with theory. Journal of Geophysical Research 64: 1933-1944. 1959.

61. Taylor, G. S. Drainable porosity evaluation from outflow measurements and its use in draw down equations. Soil Science 90: 338-343. 1960.

62. Thornthwaite, C. W. An approach toward a rational classification of climate. Geophysical Review 38: 55-94. 1948.

63. Toksöz, S. and D. Kirkham. Graphical solution and interpretation of a new drain-spacing formula. Journal of Geophysical Research 66: 509-516. 1961.

64. van Bavel, C. H. M. Drought and water surplus in agricultural soils of the Lower Mississippi Valley area. U.S. Department of Agriculture Technical Bulletin 1209. 1959.

65. van Bavel, C. H. M. A drought criterion and its application in evaluating drought incidence and occurence. Agronomy Journal 45: 167-171. 1953.

66. van Deemter, J. J. Results of mathematical approach to some flow problems connected with drainage and irrigation. Applied Scientific Research Section A, 2: 33-53. 1949.

67. van Schilfgaarde, J. Design of tile drainage for falling water tables. American Society of Civil Engineers Irrigation and Drainage Division Proceedings 89, IR2: 1-11. 1963.

68. van Schilfgaarde, J. Limitations of Dupuit-Forchheimer theory in drainage. Unpublished mimeographed paper presented at American Society of Agricultural Engineers Meeting, New Orleans, Louisiana, December, 1964. Raleigh, North Carolina, Department of Agricultural Engineering, North Carolina State College. ca. 1964.

69. van Schilfgaarde, J. Theory of land drainage. Agronomy 7: 79-112. 1957.

70. van Schilfgaarde, J. Transient design of drainage systems. Unpublished mimeographed paper presented at American Society of Civil Engineers Water Resources Engineering Conference, Mobile, Alabama, March, 1965. Beltsville, Maryland, Agricultural Research Service, U.S. Department of Agriculture. ca. 1965.

71. van Schilfgaarde, J., D. Kirkham, and R. K. Frevert. Physical and mathematical theories of tile and ditch drainage and their usefulness and design. Iowa Agricultural Experiment Station Research Bulletin 436. 1956.

72. Veihmeyer, F. J. and A. H. Hendrickson. Does transporation decrease as the soil moisture decreases? American Geophysical Union Transactions 36: 425-448. 1955.

73. Visser, W. C. Tile drainage in The Netherlands. Journal of Agricultural Science 2: 69-87. 1954.

74. Walker, P. Depth and spacing for drain laterals as computed from coresample permeability measurements. Agricultural Engineering 33: 71-73. 1952.

75. Wesseling, J. The relation between rainfall, drain discharge and depth of the water table in tile-drained land. Netherlands Journal for Agricultural Science 6: 47-61. 1958.

76. Wiser, E. H. and J. van Schilfgaarde. Prediction of irrigation water requirements using a water balance. International Congress of Agricultural Engineers, 6th, Proceedings 2: 121-129. 1964.

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APPENDIX A

List of Symbols

A - constant in van Schilfgaarde's equation equal to fFCS/K (T)

a - constant in Thornthwaite's equation (-)

API - antecedent precipitation index (-)

AWC - available water-holding capacity (L)

b - fixed point in time (T)

C - ratio of average flux between the drains to the flux midway between the drains (-)

c - constant in Thornthwaite's equation (-)

D - average depth of flow above the impervious layer (L)

E - evaporation (L)

ET - evapotranspiration (L)

EXCS - excess moisture (L)

e - monthly evapotranspiration in Thornthwaite's equation (-)

exp - the base of the natural logarithm raised to an exponent (-)

F - infinite series defined by Toksoz and Kirkham (-)

f - drainable porosity (-)

h - distance below tile drain to impervious layer (L)

i - index (-)

j - Kraijenhoff's reservoir coefficient (T)

K - hydraulic conductivity (L/T)

L - Laplace transform (-)

M - order of magnitude of events in recurrence interval (-)

m - integers (-)

N - Number of years in recurrence interval (T)

n - integers (-)

P, PCP - constant precipitation rate (L/T)

p(t) - variable precipitation rate (L/T)

R - rate of excess moisture (L/T)

RI - recurrence interval (T)

RNF - surface runoff (L)

r - radius of tile drain (L)

r_n - remainder of infinite series after n terms (-)

S - drain spacing (L)

SW - soil moisture content (-)

T - time variable (T)

TOT - total available moisture in root zone (L)

t - time variable (T)

U $(t-t_i)$ - unit step function (-)

Y - height of water table above axis of tile drains at a point midway between tile lines (L)

Y^* - Kraijenhoff's notation for a portion of an equation (L)

Z - drawdown of water table measured from ground surface at a point midway between tile lines (L)

α - angle of inclination between the recession curve and the time axis used to determine Kraijenhoff's reservoir coefficient (-)

β - constant used in Laplace transform

ε - infinitesimal time increment (T)

APPENDIX B

Application of the Unit Step Function in Equation 15

One of the simplest discontinuous functions is the unit step function, $U(t-t_i)$, where U is a function of time t, and t_i plays the role of a parameter which indicates the point at which a unit step begins. The function $U(t-t_i)$ possesses a Laplace transform (23, p. 62) which may be indicated as:

$$L\left\{U(t-t_i)\right\} = (e^{-t_i s})/s, \qquad 31$$

where

s = any complex number.

In problems where a physical impulse exists, a mathematical idealization may be considered where any function F(t) can be expressed as an impulse function.

$$F(t) = M(t_1-t_i)\cdot[U(t-t_i)-U(t-t_1)], \quad (t_1>t_i) \qquad 32$$

where

$t = 0$, except in a short interval $t_i<t<t_1$ and

M = the strength of the impulse function.

Since it is convenient to think of an impulse function as acting at a certain instant, Equation 31 may be adapted to the following notation:

$$F(t) = MI(t-t_i,\varepsilon), \qquad 33$$

which means that the impulse function F(t) is applied at time $t = t_i$, has a duration $\varepsilon>0$ and has strength M. Therefore, for the unit impulse function where M = 1,

$$I(t-t_i,\varepsilon) = (1/\varepsilon)\,[U(t-t_i) - U(t-\overline{t_i+\varepsilon})]. \qquad 34$$

Now, if M is permitted to take the value of Y and Y_o is equal to zero, then Equation 15 may be written as:

$$dY/dt + Y/A = Y_o/\varepsilon[U(t-t_i) - U(t-\overline{t_i+\varepsilon})] \qquad 35$$

where

t_i = starting time,

Y_o = magnitude of pulse or initial value of Y and

ε = time interval over which pulse of magnitude Y_o is applied.

Remembering that the Laplace transform of a derivative is

$$L\left\{F'(t)\right\} = sL\left\{F(t)\right\} - F(0),$$

or

$$L\left\{F'(t)\right\} = sf(s), \qquad 36$$

Equation 35 may be written as:

$$sy(s) + 1/A\; y(s) = (Y_o/\varepsilon)L\left\{U(t-t_i) - U(t-\overline{t_i+\varepsilon})\right\}. \qquad 37$$

Then

$$(s+1/A)y(s) = (Y_o/\varepsilon)\lim_{\varepsilon\to 0}\,(e^{-t_i s})/s - [e^{-(t_i+\varepsilon)}]/s$$

or

$$(s+1/A)y(s) = Y_o \lim_{\varepsilon\to 0}\,[e^{-t_i s} - e^{-(t_i+\varepsilon)s}]/s$$

and

$$(s+1/A)y(s) = Y_o\, e^{-t_i s} \qquad 38$$

The inverse is now desired, which involves an impulse Y_o at $t=t_i$.

Rearranging Equation 38,

$$y(s) = Y_o e^{-t_i s}\ (1/(s+1/A),$$

and taking the inverse,

$$Y(t) = Y_o L^{-1}\left\{e^{-t_i s}[1/(s+1/A)]\right\}. \qquad 39$$

Remember from the translation theorem (23, p.8) the relation

$$L\left\{e^{at}F(t)\right\} = f(s-a) \qquad 40$$

where f(s) is translated by replacing s by (s-a).

Since

$$L^{-1}\left\{(e^{-t_i s})/s\right\} = U(t-t_i) \qquad 41$$

and

$$L^{-1}\left\{1/(s-\beta)\right\} = e^{\beta t}, \qquad 42$$

Equation 39 reduces to

$$Y(t) = [Y_o e^{-(t-t_i)}/A]\ [U(t-t_i)]\ , \qquad 16$$

which is the solution for an impulse Y_o at $t = t_i$.

APPENDIX C

Calibration Data for Fluid Applicator

Table 13. Calibration of discharge from the manifold of the fluid applicator where the fluid was measured in ml

Run	Tube Number 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	9.0	9.1	9.0	8.7	8.8	8.9	8.9	8.9	9.1	9.0	9.0	9.0	9.2	9.0	9.2	9.3
2	9.1	9.2	9.0	8.8	8.9	8.9	9.0	8.9	9.0	9.1	8.9	8.9	9.0	8.8	9.0	9.5
3	9.6	9.8	9.6	9.2	9.4	9.5	9.6	9.5	9.5	9.6	9.5	9.4	9.6	9.4	9.5	9.8
4	9.2	9.1	9.0	8.9	9.0	9.1	9.0	9.0	9.3	9.1	8.8	9.0	9.1	8.9	9.0	9.3
5	9.3	9.2	9.0	9.0	9.2	9.1	9.1	9.2	9.6	9.7	9.3	9.1	9.3	9.1	9.0	9.5
Avg.	9.24	9.28	9.12	8.92	9.06	9.10	9.12	9.10	9.30	9.30	9.10	9.08	9.24	9.04	9.15	9.48

Table 14. Calibration of discharge from the distributor of the fluid applicator where the fluid was measured in ml

	Orifice Number				
Compartment	1	2	3	4	5
1	1.75	1.97	1.87	1.90	1.67
2	1.72	1.67	2.05	1.87	2.15
3	1.50	1.90	1.95	1.97	2.40
4	1.75	1.87	2.10	1.95	1.80
5	1.80	1.90	2.10	1.80	1.62
6	1.95	1.95	1.90	1.85	1.75
7	1.62	1.92	2.35	1.80	1.70
8	1.72	1.75	2.30	1.70	1.80
9	1.92	2.12	1.80	1.85	1.95
10	1.77	1.90	1.92	2.20	1.75
11	1.80	1.92	2.20	2.07	1.72
12	1.82	2.25	1.40	1.92	1.52
13	1.55	1.75	2.05	1.70	2.05
Avg.	1.82	1.90	1.99	1.89	1.83

APPENDIX D

Table 15. Values of F for a tile diameter of 0.25 foot, tile spacings of 40, 80, 160, and 320 feet, and depths to the impervious layer in half-foot increments from 0.5 to 4 feet and in one-foot increments from 5 to 10 feet

Tile spacing ft.	Depth to impervious layer below tile ft.	F-value	Tile spacing, ft.	Depth to impervious layer below tile ft.	F-value
40	0.50	9.77	40	4.00	1.77
80	0.50	19.55	80	4.00	3.02
160	0.50	38.95	160	4.00	5.51
320	0.50	77.55	320	4.00	10.45
40	1.00	5.06	40	5.00	1.59
80	1.00	10.01	80	5.00	2.59
160	1.00	19.78	160	5.00	4.59
320	1.00	39.18	320	5.00	8.56
40	1.50	3.54	40	6.00	1.48
80	1.50	6.85	80	6.00	2.31
160	1.50	13.41	160	6.00	3.97
320	1.50	26.39	320	6.00	7.29
40	2.00	2.80	40	7.00	1.41
80	2.00	5.29	80	7.00	2.12
160	2.00	10.23	160	7.00	3.55
320	2.00	20.01	320	7.00	6.40
40	2.50	2.37	40	8.00	1.36
80	2.50	4.36	80	8.00	1.99
160	2.50	8.33	160	8.00	3.24
320	2.50	16.18	320	8.00	5.73
40	3.00	2.09	40	9.00	1.33
80	3.00	3.75	80	9.00	1.89
160	3.00	7.07	160	9.00	3.00
320	3.00	13.63	320	9.00	5.22
40	3.50	1.90	40	10.00	1.31
80	3.50	3.33	80	10.00	1.81
160	3.50	6.18	160	10.00	2.81
320	3.50	11.81	320	10.00	4.81

COMPUTER PROGRAM FOR F VALUES IN KIRKHAM EQUATION FOR M=10

```
      WRITE(2,4)
    4 FORMAT(1H1,2X,12HR=RADIUS,FT.,2X,13HS=SPACING,FT.,2X,
     136HH=DEPTH BELOW DRAIN TO IMP. LAY.,FT.)
      WRITE(2,5)
    5 FORMAT(1H0,5X,1HR,8X,1HS,9X,1HH,8X,2HAF/)
      R=0.25
      L=0
    6 H=0.0
      DO 20 J=1,20,1
      H=H+0.5
      S=0.0
      DO 20 K=1,9,1
      S=S+40.0
      BPIE=3.14159
      CPIE=1.0/BPIE
      A=CPIE*(S/R)
      AL=LOGF(A)
      ALP=AL*CPIE
      M=0
      AF=0.0
      SUM=0.0
      DO 10 M=1,10
      BM=M
      CM=1.0/BM
      RSMP=BM*R*(BPIE/S)*2.0
      SOC1=COSF(RSMP)
      EM=BM*BPIE
      SOC2=COSF(EM)
      HSMP=BM*H*(BPIE/S)*2.0
      HNAT=TANHF(HSMP)
      HTOC=1.0/HNAT
      DIF1=SOC1-SOC2
      DIF2=HTOC-1.0
      SUMM=CM*DIF1*DIF2
      SUM=SUM+SUMM
   10 CONTINUE
      SEC=CPIE*SUM
      AF=ALP+SEC
      WRITE(2,15)R,S,H,AF
   15 FORMAT(1H ,1X,F6.3,2X,F8.2,2X,F8.2,2X,F8.4)
   20 CONTINUE
      L=L+1
      GO TO (21,22),L
   21 R=0.166
      GO TO 6
   22 CONTINUE
      END
```